Supplementary Material for: Identifying Firm-Level Financial Frictions using Theory-Informed Restrictions (For Online Publication)

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Abstract

This supplement contains:

- S1: Proof of Proposition 1
- S2: Extended model derivations
- S3: Econometric details
- S4: Simulation Study
- S5: More empirical results

S1 Proofs

Proof of Proposition 1

Here we prove the sign restrictions in (16) and the magnitude restrictions in (17). From the budget constraint (2) we obtain:

$$\Theta(b_t, \xi_t) = wl_t + \mathrm{div}_t - (s_t - \theta_t F) \tag{S1}$$

The net funds borrowed, $\Theta(b_t, \xi_t)$, cover the difference between financial wealth net of the overhead costs, $s_t - \theta_t F$, and the expenditures for wages wl_t and dividends div_t. As discussed in Section 2.1, the discount term $\mu > 1$ is an incentive for the firm to borrow to distribute dividends. It follows that in the steady state the firm is a net borrower, and $\Theta(b_t, \xi_t)$ is strictly positive. To see this, consider a firm that is unconstrained, and is able to finance its wage bill with internal finance, without borrowing. In this case, keeping one unit of resources as savings generates $1 + r$ units next period, and hence its net present value is $(1 + r)(\frac{1}{1+r}\frac{1}{\mu}) = \frac{1}{\mu} < 1$. Since the value to distribute this unit as dividends in the current period t is equal to 1, it follows that the firm finds it optimal to distribute all its savings as dividends. Then the firm will start to borrow to increase dividends further, and in doing so its cost of borrowing increases above $1 + r$, because of financing frictions, until it becomes so large that the firm will stop increasing dividends.

Furthermore, notice that the functional form in (4) implies that the elasticity of the amount borrowed $\Theta(b_t, \xi_t)$ with respect to the financial frictions process ξ_t is negative. Moreover, the function $\Theta(b_t, \xi_t)$ is concave and increasing in b_t, and reaches the maximum value for $b_t = b_t^M \equiv \left[\frac{1}{1+r}\frac{1}{\gamma\xi_t}\right]^{\frac{1}{\gamma-1}}$. The firm will never optimally choose a face value of debt larger than b_t^M because it would imply increasing the promised payment next period to the creditors while actually receiving less funds today. Therefore, a necessary condition for the optimal choice of debt is:

$$\frac{\partial\Theta_t}{\partial b_t}\frac{b_t}{\Theta_t} > 0 \ . \tag{S2}$$

We log-linearize $\Theta(b_t, \xi_t)$ and obtain

$$\log \Theta_t = \pi_b \log b_t - \pi_\xi \log \xi_t \ , \tag{S3}$$

where the above discussion clarifies that that $\pi_b > 0$ and $\pi_\xi > 0$.

Before log-linearising the budget constraint, we need to define the optimal dividend policy.

In equilibrium the firm decides dividends balancing the need to distribute earnings early, because of the high discount factor, with the need to save to accumulate wealth and reduce future financial frictions. Conditional on current wealth s_t, a sufficient statistic for the opportunity cost to distribute dividends rather than saving is the shadow cost of finance ψ_t. For example, if the firm is suddenly more productive and needs to increase production inputs, this will increase ψ_t in equilibrium signalling an higher cost of distributing dividends and hence reducing available resources to invest. Therefore, conditional on current financial wealth s_t, an increase in the shadow cost of finance ψ_t implies the firm desires to reduce dividends. Hence a log-linear approximation of the dividend policy can be derived as follows:

$$\log \mathrm{div}_t = \pi^s_{div} \log s_t - \pi_{div} \log \psi_t \ , \tag{S4}$$

where $\pi_{div} \geq 0$ and $\pi^s_{div} \geq 0$.These elasticities are equal to zero when the dividend constraint (3) is binding, otherwise they are positive. Log-linearising (S1) around the steady state and using (S3) and (S4) yields:

$$\pi_b \log b_t - \pi_\xi \log \xi_t = - \left(\frac{s - \overline{div}\pi^s_{div}}{\Theta} \right) \log s_t + \frac{F}{\Theta} \log \theta_t + \frac{wl}{\Theta} \log l_t - \overline{div}\pi_{div} \log \psi_t \ ,$$

where $\overline{div}$ is the (positive) steady state value of dividends, and as discussed above, Θ is positive. We substitute $\log l_t$ and $\log b_t$ using (10) and solve for $\log \psi_t$. The comparison with Equation (11) delivers the solution for the coefficients π_2, π_3 and π_4:

$$\pi_2 = \frac{\pi_b \frac{1}{\gamma-1} + \pi_\xi}{\pi_b \ \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta}} \ , \qquad \pi_3 = \frac{\frac{F}{\Theta}}{\pi_b \ \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta}}$$

$$\text{and} \qquad \pi_4 = \frac{\frac{wl}{\Theta}\frac{1}{1-\alpha}}{\pi_b\ \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta}} \tag{S5}$$

Assumptions 1 and 2 together with the fact that π_ξ and π_b are positive constants, that π_{div} is non-negative, and that $\psi > 1$, imply that π_2 and π_3 are positive coefficients, while π_4 is between 0 and 1. This proves all the magnitude restrictions in (17) and all the sign restrictions in (16) except the fact that the element B_{11} in the matrix B is negative.

The term B_{11} is equal to $-(\frac{1}{\gamma-1} - \frac{\psi\pi_2}{(\gamma-1)(\psi-1)})$. $\frac{1}{\gamma-1}$ is the direct negative effect of the ξ_t shock on borrowing b_t. However, reducing borrowing reduces labour input, and this increases financial frictions ψ_t (see the labour first order condition 9) driving the firm to increase borrowing, and partly dampening the direct effect of the shock. The dampening factor is the term $\frac{\psi\pi_2}{(\gamma-1)(\psi-1)}$. Intuitively, since the marginal product of labour has increased, the firm finds it optimal to borrow more until the higher marginal cost of external finance compensates the higher return on labour. Therefore, this equilibrium implies that borrowing is always lower than before the shock (otherwise labour input would not decrease), meaning that the optimality conditions imply that the dampening factor $\frac{\psi\pi_2}{(\gamma-1)(\psi-1)}$ cannot be larger than the direct effect $\frac{1}{\gamma-1}$, and hence the term B_{11} is negative. This proves Proposition 1.

S2 Model extensions

S2.1 Static model with endogenous convex excess cost of finance

In this section we provide a microfoundation of the convex cost of external finance assumed in (4). Consider the following simplified static version of our benchmark model. The firm borrows $\hat{b} = \dfrac{b}{1+r} - c$ to finance the labour input and produce $y = zl^\alpha$ at the beginning of the second period. r is the real interest rate while we assume the wage is $w = 1$ and as in our main model, the term c measures the excess cost of financial constraints. The timing is as follow. Before observing z, the firm needs to borrow to be able to hire workers. Since borrowing is costly, the firm only borrows the amount necessary to pay wages, so that $\hat{b} = l$, and profits net of repaying the debt are:

$$\pi = z\, \hat{b}^{\alpha} - b \tag{S6}$$

After borrowing and before actually hiring labour, the firm has $\hat{b}$ amount of cash. It observes z, drawn from a given distribution with $z \in [0, Z]$, and then has two options: i) hire labour, produce, repay the debt and obtaint the profit π. ii) default and steal a fraction λ of the cash. In this case we assume the lenders get zero return, because any additional cash is lost. The firm defaults if the return from defauting is higher than producing and repaying the debt, namely if $\lambda\hat{b} > \pi$, while it chooses to produce and repay the debt if $\lambda\hat{b} \leq \pi$. Substituting π using Equation S6 and rearranging, we determine that the minimum productivity not to default, denoted with z^* is:

$$z^* = \frac{\lambda\hat{b} + b}{\hat{b}^{\alpha}}. \tag{S7}$$

Ex ante, a risk neutral lender which requires an expected return of $1+r$, will lend the amount $\hat{b}$ under the promise of a repayment of b, such that the expected repayment $p\,(z > z^*)\, b$ is:

$$p\,(z > z^*)\, b = (1+r)\hat{b}. \tag{S8}$$

The excess cost of borrowing can be defined as:

$$c = \frac{b}{1+r} - \hat{b}, \tag{S9}$$

that is, the difference between the sum the firm would receive in the absence of financial imperfections $\dfrac{b}{1+r}$ and $\hat{b}$. Substituting $\hat{b}$ in (S8) using (S9) yields:

$$c = \frac{b}{R^F}\left(1 - p\,(z > z^*)\right)$$

if $p\,(z > z^*)$ was constant, then c would be linear in the face value of debt b. However, from (S6) and (S7) it is easy to see that an increasing in borrowing $\hat{b}$ increases z^*, reduces $p(z > z^*)$ and increases b, implying that c is increasing and convex in b.

S2.2 Derivation of the model with capital

In this section, we prove that Proposition 1 continues to hold for the linearized model with capital that was discussed in Section 2.3.1. The first order condition for debt is unchanged and the one for labour becomes

$$\frac{\alpha z_t k_t^{\beta}}{l_t^{1-\alpha}} = (1+r)\,\psi_t w \ . \tag{S10}$$

Furthermore, as in the benchmark model, $\log \psi_t$ can be approximated by a linear function:

$$\log \psi_t = \pi_1 \log s_t + \pi_2 \log \xi_t + \pi_3 \log \theta_t + \pi_4 \log z_t - \pi_5 \log k_t \ .^{1} \tag{S11}$$

By substituting it back into the log linearized equations and solving for the reduced form we obtain

$$Y_t = c + DW_t + Bg_t \ ,$$

Where $W_t = (s_t, k_t)'$, while the B matrix is identical to the one in the benchmark model. The main difference is that the budget constraint is now determined by Equation (20), where:

$$\log \Theta_t = \pi_b \log b_t - \pi_\xi \log \xi_t + \pi_k \log k_t$$

[1] Note that the proof of proposition 1 does not require any assumption regarding the coefficient π_5. Such coefficient is likely positive, because more capital increases the assets available to the firm. However, it might be negative in certain situations if capital is subject to substantial disinvestment costs. Consider for example the extreme case that capital is fully irreversible. In this case, it could be that after a negative productivity shock the firm is forced to hold an inefficiently high amount of capital which increases the intensity of financial frictions. On the one hand, this possibility is unlikely to be empirically very relevant, and any bias would directly affect only the D matrix. On the other hand, one could allow for a more flexible relationship between $\log \psi_t$ and k_t, and this would again only affect the matrix D, not the identification of B.

As in the benchmark case, $\pi_b > 0$ and $\pi_\xi > 0$, while we do not need to impose any restriction on π_k. Log-linearising the budget constraint following the same procedure outlined in Appendix A1, using (21), and solving for $\log \psi_t$ yields:

$$\begin{gathered}\pi_2 = \frac{\pi_b \frac{1}{\gamma-1} + \pi_\xi}{\pi_b \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta} + \frac{\epsilon^i_\psi}{\Theta}};\, \pi_3 = \frac{\frac{F}{\Theta}}{\pi_b \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta} + \frac{\epsilon^i_\psi}{\Theta}}; \\ \pi_4 = \frac{\frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\epsilon^i_z}{\Theta}}{\pi_b \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta} + \frac{\epsilon^i_\psi}{\Theta}}\end{gathered} \tag{S12}$$

These coefficients are identical to those derived in the benchmark case (see Equation S5) except for the presence of the term $\frac{\epsilon^i_\psi}{\Theta}$ at the denominator and of the term $\frac{\epsilon^i_z}{\Theta}$ at the numerator of π_4. Both terms are weakly positive, as shown in (22), and moreover ϵ^i_z, the elasticity of investment to productivity, is likely to be small if positive, because of capital adjustment costs, so that π_4 continues to be smaller than 1, which is consistent with the fact that it represents a dampening factor. If it was larger than 1, it would imply that positive productivity shocks increase financial frictions so much that the firm actually wants to reduce its variable input l_t, which would violate profit maximization. It is then easy to see that the proof of Proposition 1 is the same as for the benchmark case.

S2.3 Models with collateralised borrowing

Asset based borrowing

Following the specification in Section 2.3.2, in the case of asset based borrowing the firm maximises (19) subject to (20), (21), (22) and (23). Taking the first order condition for debt, assuming λ^1_t is constant and log-linearising yields:

$$\log b_t = \frac{1}{\gamma-1}\frac{\psi}{\psi-1}\log\psi_t - \frac{1}{\gamma-1}\log\xi_t + \frac{\gamma}{\gamma-1}\log k_t \; .$$

The first order condition for labour is the same as (S10) and $\log\psi_t$ is determined by (S11). We can substitute and solve for $\log b_t, \log l_t$ and $\log y_t$, and verify that the B Matrix is the same as in the benchmark model.

Therefore, it is possible to prove Proposition 1 following the same procedure outlined above. Intuitively, more capital k_t relaxes the borrowing constraint, but since it is predetermined, the only parameter affected is π_5 the other parameters π_2, π_3 and π_4 are identical to those derived above in Section S1.

Financial shock to the collateral value of assets

If we assume that the financial shock is a reduction in $1 - \lambda_t^1$, the collateral value of capital, rather than an increase in ξ_t, then the linearized first order condition for the debt becomes:

$$\log b_t = \frac{1}{\gamma - 1}\frac{\psi}{\psi - 1} \log \psi_t - \frac{\gamma}{\gamma - 1} \log(1 - \lambda_t^1) + \frac{\gamma}{\gamma - 1} \log k_t$$

Following the same procedure outlined above, the system becomes:

$$D_s = \begin{bmatrix} -\frac{1}{\gamma-1}\frac{\psi}{\psi-1}\pi_1 \\ \frac{1}{1-\alpha}\pi_1 \\ \frac{\alpha}{1-\alpha}\pi_1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -\left(\frac{\gamma}{\gamma-1} - \frac{1}{\gamma-1}\frac{\psi}{\psi-1}\pi_2\right) & \frac{\pi_3}{(\gamma-1)(\psi-1)} & \frac{\pi_4}{(\gamma-1)(\psi-1)} \\ -\frac{\pi_2}{1-\alpha} & -\frac{\pi_3}{1-\alpha} & \frac{1-\pi_4}{1-\alpha} \\ -\frac{\alpha\pi_2}{1-\alpha} & -\frac{\alpha\pi_3}{1-\alpha} & \frac{1-\alpha\pi_4}{1-\alpha} \end{bmatrix}.$$

Which is identical to the benchmark case except for the B_{11} coefficient, whose first term is $-\frac{\gamma}{\gamma-1}$ instead of $-\frac{1}{\gamma-1}$. Notice that, in this case, the last term in Equation 12 would be $g_t = \log(1 - \lambda_t, \theta_t, z_t)'$. This difference does not alter the properties of the system and the proof of proposition 1 is once agan identical to the one outlined above. In other words, our approach is able to capture any type of financial shocks, both those that increase the cost of credit and those that reduce its quantity.

Models with collateralised borrowing: Earnings based borrowing

Using the new definition of financial costs in (24), the firm maximises the value function subject to

$$div_t = s_t - F\theta_t - wl_t + \frac{b_t}{1 + r} - \xi_t \left[\frac{b_t}{(1 - \lambda_t^2)\,\pi_t}\right]^\gamma .$$

Where π_t is defined in (25). Note that, also in this case, a positive shock to the cost of credit ξ_t and a negative shock to quantity of collateral λ_t^2 have similar implications in a linear approximation of this model. Therefore we consider the shock to be ξ_t and we assume λ_t^2 to be constant. Taking the first order condition for labour, and using (7) and the definition of π_t we obtain:

$$\frac{\alpha z_t l_t^{\alpha-1}}{1+r}\frac{1}{\psi_t} - w = -\left(\frac{1}{1-\lambda^2}\right)^{\gamma} \gamma \xi_t b_t^{\gamma} \pi_t^{-\gamma-1}\left(\frac{\alpha z_t l_t^{\alpha-1}}{1+r} - w\right) \tag{S13}$$

We evaluate this first order condition in the steady state:

$$\frac{\alpha z l^{\alpha-1}}{1+r}\frac{1}{\psi} - w = -\left(\frac{1}{1-\lambda^2}\right)^{\gamma} \gamma \xi b^{\gamma} \pi^{-\gamma-1}\left(\frac{\alpha z l^{\alpha-1}}{1+r} - w\right) \tag{S14}$$

In the benchmark case we have shown that $\dfrac{\alpha z l^{\alpha-1}}{1+r}\dfrac{1}{\psi} - w = 0$. Since $\psi > 1$, it follows that the last term on the right hand side of (S14), $\left(\dfrac{\alpha z l^{\alpha-1}}{1+r} - w\right)$, is positive. This term represents the increase in profits obtained by increasing labour input in a financially constrained equilibrium. Hence the right hand side of equation (S14) has negative value, meaning that the earning based borrowing reduces the effective cost of labour relative to the benchmark case, increasing labour demand. Intuitively, labour has an additional value in increasing profits, and the firm chooses optimally a level above the level chosen in the benchmark model, but still below the level in the absence of financial frictions.

We derive the first order condition for debt to obtain:

$$b_t = \frac{1}{1+r}\left(\frac{y_t}{1+r} - w l_t\right)^{\frac{\gamma}{\gamma-1}} (1-\lambda_t^2)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\gamma \xi_t}\right)^{\frac{1}{\gamma-1}} \left(1 - \frac{1}{\psi_t}\right)^{\frac{1}{\gamma-1}}.$$

And we substitite it in (S13) and rearrange obtaining:

$$\frac{\alpha z_t l_t^{\alpha-1}}{1+r}\frac{1}{\psi_t} = w - \Psi_t$$

where

$$\Psi_t \equiv (\gamma\xi_t)^{-\frac{1}{\gamma-1}} \left(\frac{\alpha z_t l_t^{\alpha-1}}{1+r} - w\right) \left(\frac{1}{1+r}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{y_t}{1+r} - wl_t\right)^{\frac{1}{\gamma-1}} \left(1-\lambda_t^2\right)^{\frac{\gamma}{\gamma-1}} \left(1-\frac{1}{\psi_t}\right)^{\frac{\gamma}{\gamma-1}} \tag{S15}$$

We then log linearise the labour first order condition around the steady state:

$$\log z_t - (1-\alpha)l_t - \log\psi_t = -\frac{1}{w-\Psi}\left(\epsilon_l^\Psi \log l_t + \epsilon_\xi^\Psi \log \xi_t + \epsilon_\psi^\Psi \log \psi_t\right) \tag{S16}$$

where $\frac{1}{w-\Psi}$ is positive, being equal to the steady state value of the marginal productivity of labour. The definition of Ψ_t in (S15) implies that, around the steady state, the elasticities of Ψ_t to ξ_t and ψ_t, $\epsilon_\xi^\Psi$and ϵ_ψ^Ψ, respectively, are positive, while the elasticity of of Ψ_t to l_t, ϵ_l^Ψ, can be positive or negative. On the one hand the term $\left(\frac{y_t}{1+r} - wl_t\right)^{\frac{1}{\gamma-1}}$ increases in l_t, because, as mentioned before, profits $\frac{y_t}{1+r} - wl_t$ increase in labour around the steady state. However, then term $\frac{\alpha z_t l_t^{\alpha-1}}{1+r} - w$ decreases in labour, because of decreasing marginal returns. Intuitively the positive term dominates when the firm is very constrained and labour input is substantially below its unconstrained optimal level. However, as l_t increases towards such level, eventually Ψ_t goes to zero, and hence the elasticity ϵ_l^Ψ becomes negative.

Rearranging we get:

$$\log l_t = \frac{1}{(1-\alpha)-\frac{1}{w-\Psi}\epsilon_l^\Psi}\log z_t + \frac{\frac{1}{w-\Psi}\epsilon_\xi^\Psi}{(1-\alpha)-\frac{1}{w-\Psi}\epsilon_l^\Psi}\log\xi_t - \frac{1-\frac{1}{w-\Psi}\epsilon_\xi^\Psi}{(1-\alpha)-\frac{1}{w-\Psi}\epsilon_l^\Psi}\log\psi_t \tag{S17}$$

Consider the elasticity of labour to the productivity shock, $\frac{1}{(1-\alpha)-\frac{1}{w-\Psi}\epsilon_l^\Psi}$. As in the benchmark model, it is larger the larger is α, which reduces the degree to which labour productivity falls when labour demand increases. A positive value of ϵ_l^Ψ means more labour input reduces its effective cost by relaxing the borrowing constraint. However, this positive effect vanishes, and eventually ϵ_l^Ψ becomes negative, as labour increases, as explained above, so that the elasticity of labour to the productivity shock is always positive and finite. Furthermore, consider the elasticity of labour to ψ_t. In the term at the numerator, $1-\dfrac{1}{w-\Psi}\epsilon_{\psi}^{\Psi}$, "1" is the direct negative effect of tighter financial frictions on labour input. "$-\dfrac{1}{w-\Psi}\epsilon_{\psi}^{\Psi}$" is the dampening effect representing the fact that, around the steady state, more labour input relaxes the borrowing constraint. As explained above, this dampening factor vanishes as labour approaches the unconstrained level, so that it is never larger than the direct effect. Otherwise, more financial frictions would not affect labour negatively, and hence the firm would not be financially constrained in equilibrium.

Finally, we log-linearise the first order condition for debt, we substitute labour using (S17), and we rearrange obtaining:

$$\log b_t = \left(\frac{\Xi}{1 - \frac{1}{w-\Psi}\epsilon_{\xi}^{\Psi}} + \frac{\gamma}{\gamma-1}\frac{\frac{zl^{\alpha}}{1+r}}{\frac{zl^{\alpha}}{1+r} - wl} \right) \log z_t - \left(\frac{1}{\gamma-1} - \Xi \right) \log \xi_t + \left(\frac{1}{\gamma-1}\frac{\psi}{\psi-1} - \Xi \right) \log \psi_t \tag{S18}$$

$$\Xi \equiv \eta \frac{\gamma}{\gamma-1}\frac{1 - \frac{1}{w-\Psi}\epsilon_{\xi}^{\Psi}}{(1-\alpha) - \frac{1}{w-\Psi}\epsilon_{l}^{\Psi}}; \eta \equiv \frac{\alpha\frac{zl^{\alpha-1}}{1+r} - w}{\frac{zl^{\alpha-1}}{1+r} - w} \tag{S19}$$

The term η mesures the elasticity of profits to labour which, as argued above, is positive in the steady state. Therefore, following the results shown above, the term Ξ is also positive, and it represents the effect labour input has, because it relaxes the borrowing constraint, on borrowing. It is easy to see that it increases the elasticity of b_t to the productivity shock, while it dampens the elasticities to both ξ_t and ψ_t. As explained before, for financial frictions to exist in the linerised equilibrium, this dampening effect cannot be larger than the direct effects. To see this, notice that the term η is only positive when the firm is financially constrained, and goes to zero as labour input approaches the unconstrained level. So the dampening effect Ξ cannot become sufficiently large to completely offset the direct effect and neutralise financial frictions.

It follows that we can represent the linear system (10) as:

$$\log b_t = \epsilon^b_z \log z_t - \epsilon^b_\xi \log \xi_t + \epsilon^b_\psi \log \psi_t \tag{S20}$$

$$\log l_t = \epsilon^l_z \log z_t + \epsilon^l_\xi \log \xi_t - \epsilon^l_\psi \log \psi_t$$

$$\log y_t = \log z_t + \alpha \log l_t$$

Where the coefficients $\epsilon^b_z, \epsilon^b_\xi, \epsilon^b_\psi, \epsilon^l_z, \epsilon^l_\xi$ *and* ϵ^l_ψ are all positive. Substituting $\log \psi_t$ using (11) we can derive the matrix B for the case of earning based borrowing:

$$B = \begin{bmatrix} -\left(\epsilon^b_\xi - \epsilon^b_\psi \pi_2\right) & \epsilon^b_\psi \pi_3 & \left(\epsilon^b_\psi \pi_4 + \epsilon^b_z\right) \\ -\left(\epsilon^l_\psi \pi_2 - \epsilon^l_\xi\right) & -\epsilon^l_\psi \pi_3 & \left(\epsilon^l_z - \epsilon^l_\psi \pi_4\right) \\ -\alpha\left(\epsilon^l_\psi \pi_2 - \epsilon^l_\xi\right) & -\alpha\epsilon^l_\psi \pi_3 & \left(1 + \alpha\left(\epsilon^l_z - \epsilon^l_\psi \pi_4\right)\right) \end{bmatrix} \tag{S21}$$

There are three terms, $\epsilon^b_\xi - \epsilon^b_\psi \pi_2$, $\epsilon^l_\psi \pi_2 - \epsilon^l_\xi$ and $\epsilon^l_z - \epsilon^l_\psi \pi_4$ in wich a direct effect (the first term), is dampened by an indirect effect (the second term). It is easy to see that this dampening effect is smaller than the direct one, for financial frictions to exist around the steady state. For the term $\epsilon^b_\xi - \epsilon^b_\psi \pi_2$, the reasoning is analogous to the one illustrated in Section S1. For the other two terms, we start by claiming that both π_2 and π_3 are positive.

For the term $\epsilon^l_\psi \pi_2 - \epsilon^l_\xi$, the first component $\epsilon^l_\psi \pi_2$ implies that a financial friction shock makes borrowing more costly, and this reduces the demand for labour, as in the benchmark model. However, the same shock also increases the relative desirability of labour input to relax the constraint, which is the second term ϵ^l_ξ. This second term is dampening in nature, meaning that it cannot be larger than the direct effect. To see this, notice that if that was the case, then financial frictions would not reduce labour, which would imply $\epsilon^\Psi_\xi = 0$, and therefore also $\epsilon^l_\xi = 0$. Hence it must be that $-\left(\epsilon^l_\psi \pi_2 - \epsilon^l_\xi\right) < 0$. A similar reasoning can be used to argue that the term $\epsilon^l_z - \epsilon^l_\psi \pi_4$ is positive, where ϵ^l_z is the direct positive effect of productivity on labour input, while $-\epsilon^l_\psi \pi_4$ is the dampening effect caused by higher financial frictions ψ_t. Once again this indirect effect cannot dominate, because it would drive the value of ϵ^l_ψ to zero. Given this, to prove Proposition 1 it is sufficient to prove that the coefficients π_2, π_3 are positive, and that and π_4 is not smaller than $-\frac{\epsilon_z^b}{\epsilon_\psi^b}$.

The budget constraint follows equation (S1), where $\Theta(b_t, \xi_t)$ includes the earning based constraint as defined in (24). We linearise $\Theta(b_t, \xi_t)$ obtaining:

$$\log \Theta_t = \pi_b \log b_t - \pi_\xi \log \xi_t + \pi_z \log z_t + \pi_l \log l_t$$

Where π_b and π_ξ are positive, as in the benchmark model, while the previous discussion implies that around the steady state more productivity z_t and more labour input allow more borrowing, and hence also π_z and π_l are positive. Therefore linearising the budget constraint yields:

$$\pi_b \log b_t - \pi_\xi \log \xi_t + \pi_z \log z_t = -(\frac{s}{\Theta} - \overline{div}\pi_{div}^s) \log s_t + \frac{F}{\Theta} \log \theta_t - \frac{\overline{div}\pi_{div}}{\Theta} \log \psi_t + \left(\frac{wl}{\Theta} - \pi_l\right) \log l_t \tag{S22}$$

Where $\dfrac{wl}{\Theta} - \pi_l$ is overall positive, otherwise labour input would not be subject to financial frictions.[2] We then proceed as before, we substitute the labour and debt policy functions in (S22), we solve for ψ_t, and we compare it to (11) to obtain a solution for the undetermined coefficients π_2, π_3 and π_4:

$$\pi_2 = \frac{\pi_b \epsilon_\xi^b + \pi_\xi + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\xi^l}{\pi_b \epsilon_\psi^b + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\psi^l + \frac{\overline{div}\pi_{div}}{\Theta}}; \quad \pi_3 = \frac{\frac{F}{\Theta}}{\pi_b \epsilon_\psi^b + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\psi^l + \frac{\overline{div}\pi_{div}}{\Theta}};$$
$$\pi_4 = \frac{\left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_z^l - \left(\pi_b \epsilon_z^b + \pi_z\right)}{\pi_b \epsilon_\psi^b + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\psi^l + \frac{\overline{div}\pi_{div}}{\Theta}}$$

It is immediate to see that $\pi_2 > 0$ and $\pi_3 > 0$. Regarding π_4, while its denominator is positive, its numerator depends on a positive term, $\left(\frac{wl}{\Theta} - \pi_l\right)$, which reflects the fact that more productivity implies the need for funds to purchase more inputs, and a negative term $-\left(\pi_b \epsilon_z^b + \pi_z\right)$, which represents the additional borrowing capacity generated by higher profits (for given inputs) when z_t increases. Intuitively, the negative term is more likely to

[2]Because the increase in wage bill would be lower than the increase in borrowing generated by relaxing the borrowing constraint.

dominate the smaller is the elasticity α, which reduces the response of inputs demand to z_t. To see this, consider that π_4 is positive at the steady state if and only if a productivity shock increases pledgeable profits more than the wage bill, or $\frac{\partial wl}{\partial z} > (1-\lambda^2)\frac{\partial \pi}{\partial z}$. We assume for simplicity that the firm is not very financially constrained in the steady state, so that Ψ is small and we can approximate to zero $\frac{\partial \Psi}{\partial z}$. Then it is possible to show that $\frac{\partial wl}{\partial z} > \frac{\partial \pi}{\partial z}$, which is a necessary and sufficient condition for $\pi_4 > 0$, requires:

$$\frac{w}{w-\Psi}\left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}} > \frac{1-\lambda^2}{2-\lambda^2}\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \tag{S23}$$

In the extreme case that Ψ is close to zero and profits are fully collateralisable ($1-\lambda^2 = 1$), π_4 is positive as long as α is larger than 0.5. However, our identifying restrictions are satisfied by much lower values than that. First, because not all profits might be pledged as collateral. For example Drechsel and Kim (2022), in order to match the debt to output ratio observed in the US, using a model with inelastic labour, consider a value of $1-\lambda^2$ equal to 0.53, which would lower the minimum value of α compatible with $\pi_4 > 0$ to 0.35. Second, condition (S23) is satisfied for a lower value of α the tighter financial frictions are, in the form of higher values of Ψ and/or ψ . Third and more importantly, our identifying conditions are satisfied also when also when π_4 is negative, as long as $\pi_4 > -\frac{\epsilon_z^b}{\epsilon_\psi^b}$, where, from (S18), it is easy to see that $-\frac{\epsilon_z^b}{\epsilon_\psi^b} < -1$. In other words, α would have to be so low that a 1% increase in productivity generates so much additional borrowing that it reduces the intensity of financial frictions more than 1% . Summing up, the term B_{13} in (S21) would be negative, thus violating proposition 1, only if the elasticity of variable inputs to output (α in our model) were extremely low, and hence most of the production was done with fixed inputs, which is unrealistic in virtually all industries. Therefore in practice our Assumption 1 is sufficient to satisfy our identification restrictions for realistically chosen values of $\underline{\alpha}$ and $\overline{\alpha}$.

S3 Some econometric details

Limiting distribution panel GMM estimates

In this section we discuss the limiting distribution of the reduced form estimates $\hat{\mu}$. We first introduce some additional notation. The commutation matrix $\mathcal{K}_{m,n}$ is defined such that, for any $(m \times n)$ matrix G; $\mathcal{K}_{m,n}\text{vec}(G) = \text{vec}(G')$; and the $m^2 \times (m(m+1))/2$ duplication matrix $\mathcal{D}_m$ is defined such that $\mathcal{D}_m\text{vech}(F) = \text{vec}(F)$ for a symmetric $(m \times m)$ matrix F: Furthermore, $\mathcal{D}_m^+ = (\mathcal{D}_m'\mathcal{D}_m)^{-1}\mathcal{D}_m'$ and $\mathcal{L}_m$ is the $(m(m+1))/2 \times m^2$ elimination matrix defined such that, for any $(m \times m)$ matrix F; $\text{vech}(F) = \mathcal{L}_m\text{vec}(F)$

To derive the joint limiting distribution of $\hat{\phi}$ and $\text{vech}(\widehat{\Sigma})$ we impose the following assumptions on the reduced form errors $u_{i,t} = B\varepsilon_{i,t}$ and the initial conditions.

Assumption S1. *We assume that $u_{i,t}$ is independently and identically distributed across i and t and there exists some $\delta > 0$ such that*

1. $\mathbb{E}_F(u_{i,t}|W_{i,t}, \ldots, W_{i,1}, Y_{i,t-1}, \ldots, Y_{i,t-p}) = 0$
2. $\mathbb{E}_F(u_{i,t}u_{j,s}'|W_{i,t}, , \ldots, W_{i,1}, Y_{i,t-1}, \ldots, Y_{i,t-p}) = \begin{cases} \Sigma & \text{if} \quad i = j,\ s = t \\ 0 & \text{else} \end{cases}$.
3. $\mathbb{E}_F\|u_{i,t}\|^{4+2\delta} < \infty$
4. $\max_i \mathbb{E}_F\|X_{i,0}\|^{4+2\delta} < \infty$ *and* $\max_i \mathbb{E}_F\|c_i\|^{4+2\delta} < \infty$
5. $\Sigma_{ZX} = \text{plim}_{N\to\infty}S_{ZX}$ *and* $\Sigma_{ZZ} = \text{plim}_{N\to\infty}S_{ZZ}$ *have full column rank*
6. $\mathbb{E}_F(u_{i,t}u_{i,t}' \otimes u_{i,t}|W_{i,t}, , \ldots, W_{i,1}, Y_{i,t-1}, \ldots, Y_{i,t-p}) = 0$

where $X_{i,0} = (W_{i,-p+1}', \ldots, W_{i,0}', Y_{i,-p}', \ldots, Y_{i,-1}')'$.

To facilitate the characterization of the limiting distribution let

$$Q = \Sigma_{ZX}'\Sigma_{ZZ}^{-1}\Sigma_{ZX} \qquad \text{and} \qquad S = -\text{plim}_{N\to\infty}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(u_{i,t} - u_{i,.})(X_{i,t} - X_{i,.})' \ .$$

The asymptotic distribution of the reduced form estimates $\hat{\mu} = (\hat{\phi}', \text{vech}(\widehat{\Sigma})')'$ is summarized in the following Theorem.

Theorem S1. *Assume that the data are generated by model* (29) *and that Assumption S1 holds. We have for $N \to \infty$ that*

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} N(0, \Omega) \ , \qquad \textit{with} \qquad \Omega = \begin{bmatrix} \Omega_{\phi\phi} & \Omega'_{\phi\sigma} \\ \Omega_{\phi\sigma} & \Omega_{\sigma\sigma} \end{bmatrix} \ ,$$

where

$$\begin{aligned}
\Omega_{\phi\phi} =& \Sigma \otimes Q^{-1} \\
\Omega_{\phi\sigma} =& -\mathcal{D}_K^+(I_K \otimes S)(\Sigma \otimes Q^{-1}) - \mathcal{D}_K^+\mathcal{K}_{K,K}(I_K \otimes S)(\Sigma \otimes Q^{-1}) \\
\Omega_{\sigma\sigma} =& \mathcal{D}_K^+ \left[\frac{1}{T+1}\Lambda_{K^2} + \frac{1}{T(T+1)}(\Sigma \otimes \Sigma)(I_{K^2} \otimes \mathcal{K}_{K,K}) \right] (\mathcal{D}_K^+)' \\
&+ \mathcal{D}_K^+(\Sigma \otimes SQ^{-1}S')(\mathcal{D}_K^+)' + \mathcal{D}_K^+(SQ^{-1}S' \otimes \Sigma)(\mathcal{D}_K^+)' \\
&+ \mathcal{D}_K^+(\Sigma \otimes SQ^{-1}S')\mathcal{K}'_{K,K}(\mathcal{D}_K^+)' + \mathcal{D}\mathcal{K}_{K,K}(\Sigma \otimes SQ^{-1}S')(\mathcal{D}_K^+)'
\end{aligned}$$

and $\Lambda_{K^2} = \mathbb{V}_F(\text{vec}(u_{i,t}u'_{i,t}))$.

Proof. See Theorem 1 in Cao and Sun (2011). □

The important difference with the aggregate time series case is that the limiting covariance between the $\hat{\phi}$ and $\text{vech}(\widehat{\Sigma})$ is not equal to zero. Therefore we need to take this correlation into account when constructing the confidence sets for the structural coefficients.

The asymptotic variance can be estimated by replacing the population coefficients Σ, Q, S and Λ_{K^2} by their sample counterparts. For Σ the expression is given in equation (34), while for Q, S these are given by

$$\widehat{Q} = S'_{ZX}S_{ZZ}^{-1}S_{ZX} \qquad \text{and} \qquad \widehat{B} = -\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(u_{i,t} - u_{i,.})(X_{i,t} - X_{i,.})' \ . \tag{S24}$$

For Λ_{K^2}, some more work is required. In general, the variance matrix Λ_{K^2} depends in a complicated way on the fourth-order multivariate cumulants of $u_{i,t}$. Therefore in practice Cao and Sun (2011) suggest to estimate this matrix by

$$\widehat{\Lambda}_{K^2} = (\widehat{\Sigma} \otimes \widehat{\Sigma})(I_{K^2} + \mathcal{K}_{K,K}) \ . \tag{S25}$$

We followed there approach, noting that in our empirical study little differences are obtained when considering a more general estimate for Λ_{K^2}.

S4 Simulation study

We use the structural model described in Section 2 to simulate an artificial industry, and draw a panel of $N = 10000$ firms for $T = 10$ periods. As explained in Fn.10 in the paper, in the model derived in Section 2.1 we abstract for endogenous exit of firms, to simplify the derivations. Instead for these simulations we consider a slightly modified version of the model in which we allow for endogenous exit to happen in equilibrium. More specifically, the value function can be written as:

$$V_t^{\text{cont}}(S_t) = \max_{l_t, b_t} (1 + \phi_t) \, \text{div}_t + \frac{1}{1+r} \frac{1}{\mu} \mathbb{E}_t \left[V_{t+1} \left(S_{t+1} \right) \right] \ , \tag{S26}$$

where V_t^{cont} is the value conditional on continuing to operate in period t. The beginning of the period value $V_t(S_t)$ is instead given by

$$V_t(S_t) = d\{1(a_t \geq 0) \max \left[V_t^{\text{cont}}(S_t), a_t \right] - 1(a_t < 0) a_t\} + (1-d) a_t \ , \tag{S27}$$

With probability $1 - d$, the firm's technology becomes useless, and the firm liquidates its activity.[3] With probability d the firm is still productive, but it might go bankrupt and be forced to liquidate if it has insufficient funds to cover the fixed overhead costs of production. We define as $a \equiv s_t - \theta_t F$ the beginning of the period financial wealth after covering these

[3]Essentially we assume the stochastic process for productivity z_t is persistent with probability d and 0 permanently with probability $1 - d$.

overhead costs. In (S27), $1(a_t \geq 0)$ is an indicator function that is equal to zero if $a_t < 0$ and the firm goes bankrupt. Note that in the calibrated model the beginning of the period financial wealth $s_t = y_{t-1} - b_{t-1}$ is always positive. That is, the firm always repays the debt. Furthermore, it is also the case that a_t is almost always positive. The firms engage in precautionary saving to avoid inefficient default that would destroy the positive net present value of their business. Therefore inefficient default with $a_t < 0$ only happens in equilibrium with around 2% probability.

If $a_t \geq 0$ and $\max\left[V_t^{\text{cont}}(S_t), a_t\right] = a_t$, the firm voluntarily exits because its value net of current wealth $(V_t^{\text{cont}}(S_t) - a_t)$, is negative. This can happen for financial reasons, after a sequence of negative profits which implies excessively high levels of debt, or for efficiency reasons, if productivity becomes too low.

In order to simulate a panel data of firm, we assume that there are N perfectly competitive firms. We assume the stochastic process for productivity z to be AR(1) while ξ and θ are assumed to be i.i.d. We solve numerically the model to obtain optimal labour choice $l_t^*(s_t, \theta_t, z_t, \xi_t)$ and optimal debt $b_t^*(s_t, \theta_t, z_t, \xi_t)$. At the beginning of period t there are N firms which were active in the previous period. N^E firms exit after observing the shocks. An equal number of firms enter to keep the total number unchanged. They have identical initial endowment a_0 and initial productivity z_0. They also draw a value of the shock ξ_t. Naturally, the values a_0 and z_0 are such that new firms want to continue regardless of ξ_t.

Calibration

The parameters of the model are calibrated to match key moments related to the dynamics of productivity and financial variables at the firm level. In other words, we make sure that the simulated panel of firms is as much as possible realistic along the main dimensions of interest. We calibrate all the parameters that are important in determining the level and volatility of productivity, liquidity shocks and financial frictions shocks by matching key firm-level empirical moments. This way we ensure that the dispersion in production opportunities, financial wealth and financial frictions in the model are as much as possible realistic (with the obvious caveat that financial frictions are difficult to measure). Among these, the discount term μ affects the desire of firms to distribute dividends and ensures they are borrowers in equilibrium. The chosen value of $\mu = 1.091$ matches a target leverage of around 25%. The implied discount factor of firms is $\frac{1}{1+r}\frac{1}{\mu} = 0.877$. This relatively low value is common in models with financially constrained entrepreneurs that are calibrated to match their distribution of wealth. For example the same parameter is equal to 0.865 in Cagetti and De Nardi (2006). The productivity latent variable z is an AR(1) process with autoregressive coefficient ρ^z, while ξ and θ are i.i.d. The unconditional means of z and θ, denoted as μ^z and μ^θ , respectively, are normalised to one. ρ^z matches the autoregressive coefficient obtained from an estimated measure of revenue TFP at the firm level. The standard deviation of the productivity shock ε^z matches the cross sectional dispersion in the growth rates of labour, and more specifically the 75th percentile. The standard deviation of the liquidity shock ε^θ matches the the 75th percentile of the cross sectional dispersion in the growth rates of profits over sales. These 3 moments are computed on our dataset of Spanish firms from SABI.[4]

The mean of ξ and the standard deviation of its innovations ε^ξ are difficult to measure empirically, which is precisely the motivation for this paper. For those datasets for which interest rate spreads are available, one can use their distribution to proxy for the distribution of the excess cost of external finance. This is of course an imperfect proxy. On the one hand, empirical spreads reflect in large part also debt riskiness, and therefore are larger than the effective expected cost of debt (because with some probability the firm is going to default and not repay it). This implies that they might overestimate the actual excess cost of finance caused by financial frictions. On the other hand, as we explained before, our estimated financial frictions shock captures not only changes in the excess cost, but also changes in the tightness of the borrowing limit. In this respect, the empirical spread is likely to underestimate the cost of financial frictions. Therefore, even though the empirical spreads are poor measures of financial frictions for a given firm in a given point in time, on average

[4]To reduce in the data some types of firm heterogeneity not represented in the model, we subtract the part of growth rates explained by sector and year fixed effect and we reduce size heterogeneity by including firms between 20 and 250 employees.

these positive and negative biases partly compensate each other, so that the moments in their distribution might be informative of the moments of the distribution of financial frictions. Thus we use these moments to calibrate our parameters related to the level and volatility of the financial frictions latent variable ξ

More specifically, we calibrate the mean of ξ, μ^{ξ}, with the average spread observed on Spanish firm from credit registry data, from Gonzalez and Sy (2023). Furthermore, we calibrate the standard deviation of ε^{ξ}, σ^{ξ}, with the dispersion in spread observed for US Compustat firms from Arellano et al. (2019). Finally, we calibrate the fixed cost parameter F by matching the median ratio between profits and sales for Compustat manufacturing firms, where we measure profits with EBIT (compustat code: ebit).

The other parameters are set as follows: the interest rate r is set to 4.5%. The initial wealth of firms is set to around 80% of median wealth of incumbents. The output elasticity of labour α is set to 0.61. The curvature of the excess cost of debt γ is set to 2.[5]

Table S1: Calibration

	Parameter Value	Target	Data	Model
μ	1.091	Leverage	0.261[1]	0.22[6]
ρ^z	0.5	AR(1) coefficent of estimated RTFP	0.5[2]	0.5
σ^z	0.0275	75% percentile of labour growth	7.4%[2]	8.1%
μ^{ξ}	0.005	Average spread on bank loans	3.7%[3]	3.51%
σ^{ξ}	0.75	Cross sectional dispersion in the spread	1.1%[4]	1.16%
σ^{θ}	0.03	75% percentile of profits growth	3.07%[2]	3.18%
F	14.16	Profits over Sales	6.2%[5]	6.5%

Notes: Untargeted parameters: μ^z and μ^{θ} are normalisded to one. $\alpha = 0.61$; $\gamma = 2$; $r = 4.5\%$; $s_0 = 25$, corresponding to 80% of the median value for incumbent firms.
1: US firm level data (Source: Giroud and Mueller 2021); 2: our calculations using Spanish firm level data from SABI; 3: Spanish credit registry (Source: Gonzalez and Sy 2023); 4: Compustat (Source: Arellano et al. 2019); 5: our calculation on manufacturing firms from Compustat; 6: Leverage computed as debt over sales.

[5]In practice, μ^{ξ} , σ^{ξ} and γ jointly determine the average cost of financial frictions and their dispersion. We chose to set $\gamma = 2$, which helps obtaining a much faster convergence in the solution of the model, and as explained below set the other two parameters to match these two moments.

Simulation Results

Our first exercise is to regress the estimated structural shocks on the outcome variables, and to compare the coefficients to those obtained with the true structural shocks. In all regressions we also include the log of beginning of the period financial wealth as control variable. The results are shown in Table S2. Column 1 shows reduced form regressions using the true shocks. These are normalised in order to have similar mean and variance of the estimated shocks, so that the size of the coefficients in column 1 are comparable to those in columns 2-4. Column 1 shows that the shocks explain a substantial part of the variation in the dependent variables.[6] Columns 2-4 in each table show the same regressions, this time using the median, lower bound and upper bound values of the estimated shocks. In all three cases, the estimated shocks also explain a large part of the variation of the dependent variables. Furthermore, the estimated coefficients in columns 2-4 are quite close to those in column 1, especially for the financial frictions shock ε_t^{ξ}, which is the main objective of our analysis, and for the productivity shock ε_t^{z}.

Our second exercise is to verify the validity of our financial frictions indicator $\mathcal{I}_t^{\xi}$ defined in (26). From our model simulations we can obtain the value of ψ_t, which measures the relative shadow value of finance and therefore the intensity through which financial frictions affect the real decisions of firms. Therefore, we can measure to what extent, in a realistically calibrated industry, fluctuations in ψ_t are driven by the financial shock ε_t^{ξ}. In Columns 1 and 2 we show, using linear regressions, the share of variation in ψ_t explained by each shock. Column 1 considers the true shocks, and Column 2 the median estimated ones. They show that the most important one is by far the financial shock, which explains 73% of the ψ_t variation in Column 1, and 64.5% in Column 2. The productivity shock only explains around 10% of the variation in ψ_t. Importantly, this is not because the productivity shock is not very volatile. In fact it is the main driver of output, explaining 52% of its variation, while the financial friction shock only explains 19% of its variation.

[6] The unexplained part is due to the fact that we do not include among the regressors the lagged level of the latent productivity process z, and to a lesser extent to the approximation induced by the linearization.

Table S2: Simulation results

Output	(1) True	(2) Median	(3) Lower	(4) Upper
$\hat{\varepsilon}^{\xi}_{i,t}$	-0.0350	-0.0321	-0.0313	-0.0324
$\hat{\varepsilon}^{\theta}_{i,t}$	-0.0011	-0.0155	-0.0163	-0.0171
$\hat{\varepsilon}^{z}_{i,t}$	0.0478	0.0500	0.0533	0.0532
Constant	3.3074	3.7280	2.7086	2.7562
Obs.	80,000	80,000	80,000	80,000
R^2	0.7329	0.6114	0.6887	0.6951

Labour	(1) True	(2) Median	(3) Lower	(4) Upper
$\hat{\varepsilon}^{\xi}_{i,t}$	-0.0571	-0.0528	-0.0519	-0.0518
$\hat{\varepsilon}^{\theta}_{i,t}$	-0.0017	-0.0242	-0.0224	-0.0241
$\hat{\varepsilon}^{z}_{i,t}$	0.0405	0.0421	0.0465	0.0463
Constant	5.6170	6.1110	5.1886	5.1816
Obs.	80,000	80,000	80,000	80,000
R^2	0.7875	0.7091	0.7586	0.7546

Debt	(1) True	(2) Median	(3) Lower	(4) Upper
$\hat{\varepsilon}^{\xi}_{i,t}$	-0.4404	-0.4177	-0.4161	-0.4241
$\hat{\varepsilon}^{\theta}_{i,t}$	0.0054	0.0887	0.1185	0.1171
$\hat{\varepsilon}^{z}_{i,t}$	0.1065	0.1175	0.1244	0.1222
Constant	2.6758	2.1284	2.8343	2.1981
Obs.	80,000	80,000	80,000	80,000
R^2	0.8832	0.8571	0.8253	0.8115

Notes: The table shows the results from regressing output $\log y_t$, labour $\log l_t$ and debt $\log b_t$ on the true shocks (column (1)) and the estimated structural shocks (columns (2)-(4)). The estimates for the shocks correspond to the median (2), lower (3) and upper (4) bound of the identified set. The log of beginning of the period financial wealth is also included in all regressions as a control variable.

The third column does a similar exercise, by directly evaluating the financial frictions indicator. We consider indicators based on the 50% highest values of the three shocks ε^{ξ}, ε^{θ}, ε^{z}, and equal to zero otherwise. In the column, we report the percentage of observations that coincide with the 50% highest values of ψ. A value around 50% would indicate that the high shock observations are unable to identify firms with high ψ. This is the case of the z and θ shocks. Conversely, we find that high ξ observations include 85% of the above median observations of ψ, and 97% of the highest quartile observations of ψ (column 4). We also find very similar results if we use the alternative financial frictions indicators described in (42) and (43).

Summing up, these simulation results show that, in the context of the class of models considered, our methodology obtains useful information on the unobservable shocks, and that the indicator $\mathcal{I}_t^{\xi}$ is a reliable indicator of financial frictions. Similar results can be obtained when data is simulated from the more elaborate models discussed in Section 2.3.[7]

Table S3: Variation in ψ explained by the shocks

	% variation explained		Share $\psi > 50\%$	Share $\psi > 75\%$
	True shock	Estimated shock		
ε^{ξ}	73%	65.6%	86%	97%
ε^{θ}	0.1%	10.8%	53%	63%
ε^{z}	9.5%	9.3%	59%	41%

Notes: In Column 1 we display the R^2 obtained from regressing the relative shadow value of finance ψ_t on each of the three true shocks individually. In column 2 we do the same exercise using instead the median estimated shocks. In Column 3 we report the percentage of observations for which the indicators that select the 50% firms with highest ε^{ξ},ε^{θ} and ε^{z} (respectively in rows 1, 2 and 3) include the observations with 50% highest value of ψ. In column 3 we show the same exercise but this time with the 75% highest values of ψ.

[7] In the appendix S2.3 we show that, in a model with earning based constraints, productivity shocks are less positively related to the intensity of financial constraints, because an increase in z increases desired investment, which increases financial frictions as in the benchmark model, but also it increases the collateral value of earnings, and thus partially relaxes financial frictions. Therefore in a model with earning based borrowing, the financial frictions shock ε^{ξ} would be even relatively more important than ε^{z} in driving the fluctuations in ψ, than in the benchmark model.

Table S4: FINANCIAL FRICTIONS IN 2007 AND EMPLOYMENT CONTRACTION IN 2008. ALTERNATIVE THRESHOLDS TO COMPUTE THE FINANCIAL FRICTIONS INDICATOR.

	(1) $> 50\%$	(2) $> 75\%$ (Benchmark)	(3) $> 90\%$
$\log(l_{t-1})$	0.895***	0.896***	0.896***
	(96.684)	(99.962)	(101.229)
$\log(l_{t-1}) * Gr$	-0.003	-0.006	-0.006
	(-0.949)	(-1.577)	(-1.546)
$\mathcal{I}^{\xi}_{t-1}$	-0.002	0.003	-0.001
	(-0.489)	(0.575)	(-0.171)
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.041***	-0.067***	-0.088***
	(-3.116)	(-3.967)	(-3.441)
Constant	0.151***	0.146***	0.148***
	(9.736)	(10.087)	(10.504)
Obs.	6,710	6,710	6,710
R^2	0.858	0.858	0.858
Number of firm	506	506	506

Notes: The table studies the differential effects of the great recession on financially constrained firms. In column 1, Firm fixed effects and Sector-Year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}^{\xi}_{t-1}$ is a dummy variable equal to one for the firms with largest value of the financial friction shock $\hat{\varepsilon}^{\xi}_{t-1}$ in year $t-1$ and zero otherwise. In columns 1, 2 and 3 the threshold value of $\hat{\varepsilon}^{\xi}_{t-1}$ is the 50th, 75th and 90th percentile, respectively. Gr is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

S5 Further empirical results

In this appendix, we provide some robustness checks of the main results shown in Section 4.1. Table S4 repeats the estimation in column 1 of Table 1, but varying the threshold τ in 41. Table S5 repeats the same estimations but selecting different values of the identified set. Results are qualitatively similar regardless on whether we compute the indicator $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$ using the upper or lower bound values of the identified set for $\hat{\varepsilon}^{\xi}_{t-1}$.

Table S6 replicates the exercise in Table 1 using the large sample of spanish firms from SABI, which is representative of the population of firms above 5 employees. One limitation of this sample is that it is only available from 2000 (some years before 2000 are available, but with very limited coverage). Since we use two lags to compute the shocks, we can only include the years from 2002 to 2007 for the independent variables. We focus on a balanced sample of firms present in the whole period, for a total of 26.868 firms. Table S6 shows that the coefficient of $\mathcal{I}^{\xi}_{t-1}$ is negative and significant, indicating that over the sample period,

Table S5: Financial frictions in 2007 and employment contraction in 2008, alternative values of the identified set

	(1) lower bound value of $\hat{\varepsilon}^{\xi}_{t-1}$	(2) median value of $\hat{\varepsilon}^{\xi}_{t-1}$ (Benchmark)	(3) upper bound value of $\hat{\varepsilon}^{\xi}_{t-1}$
$\log(l_{t-1})$	0.895***	0.896***	0.895***
	(99.422)	(99.962)	(97.772)
$\log(l_{t-1}) * Gr$	-0.003	-0.006	-0.005
	(-0.845)	(-1.577)	(-1.477)
$\mathcal{I}^{\xi}_{t-1}$	-0.002	0.003	-0.004
	(-0.407)	(0.575)	(-0.627)
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.052***	-0.067***	-0.039**
	(-3.285)	(-3.967)	(-2.103)
Obs.	6,735	6,735	6,735
R^2	0.856	0.857	0.856
Number of firm	508	508	508

Notes: The table studies the differential effects of the great recession on financially constrained firms. In column 1, Firm fixed effects and Sector-Year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}^{\xi}_{t-1}$ is a dummy variable equal to one for the firms with largest value of the financial friction shock $\hat{\varepsilon}^{\xi}_{t-1}$ in year $t-1$ and zero otherwise. In columns 1, 2 and 3 the value of $\hat{\varepsilon}^{\xi}_{t-1}$ corresponds to the lower bound, median, and upper bound of the identified set, respectively. Gr is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

firms that are more financially constrained in year t reduce their employment in year $t+1$ more than the other firms. This result, which is in contrast to what we found for Compustat in Table 1, is likely due to the fact that we are analysing predominantly small firms for which financial shocks are more persistent than for large ones also in normal times, not only during a financial crisis.

Nonetheless, we also find that the Coefficient of $\mathcal{I}^{\xi}_{t-1} * Gr$ is also negative and significant in all specifications, indicating that this negative relation was even stronger between year 2007 and 2008, thus confirming the results obtained for the Compustat sample.[8]

[8] Note that, due to the data limitation explained above, the SABI sample only starts in 2002, and hence it could be that the coefficient of $\mathcal{I}^{\xi}_{t-1} * Gr$ is in part driven by the fact that we compare 2007 with a limited number of years in which there was fast growth in Spain. Conversely, for the Compustat sample, we avoid this problem by focusing on a much longer time series and by performing several placebo experiments, as explained in Section 4.1.1.

Table S6: Financial frictions in 2007 and employment contraction in 2008 - Small and Medium Spanish Manufacturing Firms.

	(1)	(2)	(3)	(4)	(5)
$\log(l_{t-1})$	0.569***	0.563***	0.576***	0.642***	0.621***
	(96.175)	(94.756)	(96.053)	(107.877)	(103.088)
$\log(l_{t-1}) * Gr$	0.002	0.000	-0.002	-0.006***	-0.007***
	(1.220)	(0.010)	(-1.487)	(-3.923)	(-4.297)
$\mathcal{I}^{\xi}_{t-1}$	-0.019***	-0.021***	-0.020***	-0.018***	
	(-15.770)	(-17.308)	(-16.780)	(-14.434)	
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.042***	-0.042***	-0.040***	-0.034***	
	(-13.134)	(-13.168)	(-12.620)	(-10.655)	
$\mathcal{I}^{Z}_{t-1}$		-0.021***	-0.016***	0.005***	
		(-17.944)	(-13.493)	(3.959)	
$\mathcal{I}^{Z}_{t-1} * Gr$		-0.037***	-0.029***	-0.011***	
		(-11.467)	(-8.779)	(-3.390)	
$Small_{t-1}$			-0.037	-0.046	-0.039
			(-0.798)	(-0.962)	(-0.803)
$Small_{t-1} * Gr$			0.004	0.002	0.001
			(1.023)	(0.567)	(0.267)
$Highlev_{t-1}$			0.001	0.000	-0.001
			(0.756)	(0.004)	(-0.460)
$Highlev_{t-1} * Gr$			-0.008**	-0.008*	-0.009**
			(-2.457)	(-1.681)	(-2.098)
$Lowprod_{t-1}$			-0.046***	-0.015***	-0.014***
			(-19.216)	(-5.396)	(-5.161)
$Lowprod_{t-1} * Gr$			-0.038***	-0.014***	-0.013***
			(-11.761)	(-3.429)	(-3.334)
$labp_{t-1}$				0.141***	0.138***
				(28.097)	(30.433)
$labp_{t-1} * GR$				0.021***	0.022***
				(7.767)	(8.361)
lev_{t-1}				-0.002	-0.015***
				(-1.221)	(-8.968)
$lev_{t-1} * GR$				-0.003	-0.004**
				(-1.395)	(-2.100)
$\hat{\varepsilon}^{\xi}_{t-1}$					-0.018***
					(-21.214)
$\hat{\varepsilon}^{\xi}_{t-1} * Gr$					-0.021***
					(-12.517)
Obs.	134,272	134,272	134,272	134,272	134,272
R^2	0.355	0.360	0.366	0.380	0.385
Number of firm	24,868	24,868	24,868	24,868	24,868

Notes: The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects and sector-year fixed effects (2-digit NACE sectors) are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}^{\xi}_{t-1}$ is a dummy variable equal to one for the upper quartile of the financial friction shock $\hat{\varepsilon}^{\xi}_{t-1}$ in year $t-1$ and zero otherwise. $\widehat{\mathcal{I}}^{Z}_{t-1}$ is a dummy variable equal to one for the lower quartile of the productivity shock $\hat{z}_{t-1}$ in year $t-1$ and zero otherwise. Gr is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. lev_{t-1} is total debt over fixed assets, and $labp_{t-1}$ is labour productivity (measured as real output divided by number of employees). The variables $highlev_{t-1}$ and $lowprod_{t-1}$, are equal to one for the 25% firm-year observations with highest leverage and lower labour productivity in year $t-1$, respectively, and equal to zero otherwise. $Small_{t-1}$ is a dummy variable if the firm belongs to the quartile of smallest firms in period $t-1$, and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

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