Identifying Firm-Level Financial Frictions using Theory-Informed Restrictions*

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Abstract

We propose a novel theory-based method for identifying financially constrained firms using firm-level balance sheet data. We separately identify firm level productivity, liquidity and financial frictions shocks from observed output, debt and production inputs using sign and magnitude restrictions. These restrictions are proven consistent with a wide range of structural dynamic models, and valid regardless of the type of financing imperfection. Based on the identified shocks we construct indicators for identifying financially constrained firms. The method is validated for large US manufacturing firms, documenting consistency with narrative methods, and for the quasi-universe of Spanish firms, highlighting its broad applicability.

Keywords: financial frictions, dynamic corporate finance theory, sign restrictions.

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1 Introduction

A large theoretical literature has shown that financial market imperfections generate a wedge between the cost of internal and external funds, and imply that firms might face financing constraints which distort their investment and employment decisions. These firm-level frictions matter for the amplification and persistence of aggregate shocks, and they explain a substantial part of the productivity gap between developing and developed countries.[1] However, despite their importance, we still have limited understanding on how to empirically identify financially constrained firms.

Existing methods can be broadly classified into three categories, which we label as theory-based methods, narrative methods, or "fixed criteria" methods. Regarding the first, several authors solve and estimate dynamic firm-level models where the latent intensity of financial frictions is an endogenous variable. A limitation of this approach is that the mapping from observables to this latent variable is inherently model-specific and generally only consistent with a specific type of financial frictions. In reality different types of firms might be subject to different types of financial constraints, and hence such theory based methods might not be able to detect all financially constrained firms.[2]

Narrative methods exploit that large public companies are often required to publish reports on their financial position, and the information in these reports can be used to identify firms that face financial frictions.[3] A limitation of this approach is that these reports are typically only available for public firms.[4] Therefore, in practice researchers often rely on ad hoc "fixed criteria", such as age, size or sector level indicators, to identify financially constrained firms. Such indicators are often only loosely related to the underlying finance theory and will generally mix identification based on financial constraints with other firm or sector level characteristics.

In this work, we propose a new theory-based approach for identifying financially constrained firms that is consistent with a wide class of models with financial imperfections and only requires commonly available balance sheet data.

The key idea underlying our method is that modern dynamic investment models with financial frictions imply optimality conditions at the firm level that come with sign and magnitude restrictions informed by canonical economic mechanisms, such as decreasing marginal

[1]For reviews of this literature, see Brunnermeier et al. (2012) and Buera et al. (2015).

[2]Heterogeneity in financial frictions is emphasised both by Nikolov et al. (2021), who show that different firms (small versus large, and private versus public) are affected by different types of financial frictions, and by the recent literature that highlights the importance of asset based and earning based borrowing constraints (e.g. Kermani and Ma 2020, Lian and Ma 2021, Caglio et al. 2021, Drechsel 2022).

[3]See for example Hoberg and Maksimovic (2014) and Buehlmaier and Whited (2018).

[4]In general, attempts to extrapolate theory-based or narrative indicators to construct quantitative financial frictions measures for the whole population (e.g. Lamont et al. 2001, Hadlock and Pierce 2010, Whited and Wu 2006) have been found to be largely unsuccessful (see Farre-Mensa and Ljungqvist 2015).

returns of variable inputs and increasing cost of debt in leverage. We show that we can exploit these restrictions to identify the underlying financial frictions, liquidity and productivity shocks in a structural panel vector autoregressive model, using similar methods as used in the macroeconometric sign-restriction literature for aggregate time series (e.g. Uhlig 2005). With the identified financial frictions shocks we can assess the effects of exogenous changes in financial frictions on firm level variables and construct indicators for identifying financially constrained firms.

The key advantage of our methodology is that, because it relies on a limited set of identifying restrictions, it is consistent with a wide class of theoretical models. More specifically, we show that the derived restrictions hold in the presence of frictions that determine a wedge between the internal and external cost of funds, as well as asset-based and earning-based borrowing constraints. In addition, the methodology addresses the other limitations of existing methods: only observed values for output, debt and production factors are required, and the resulting financial frictions shocks are firm specific and not sector level indicators for instance.

Methodology We start by developing our method for a stylized dynamic optimization problem of a firm that uses a variable input to produce and faces financial frictions that imply that the premium in the cost of external finance increases in leverage. The firm is subject to productivity shocks (which could also be interpreted as demand shocks), to liquidity shocks, that affect profits and wealth but are unrelated to productivity, and to financial shocks, which affect the premium cost of external finance. We derive the first order conditions of the model and linearize them to obtain a system of three equations in output, debt, and variable input, with the three exogenous shocks.

We show that, by imposing a minimal set of assumptions - decreasing marginal returns in the variable input, and excess cost of debt increasing and convex in leverage - it is possible to derive a set of firm-level sign and magnitude restrictions that can separately identify the three structural shocks. Importantly, these identifying restrictions continue to be sufficient in extensions of the model, e.g. in the presence of other inputs potentially subject to adjustment costs, and of asset-based and earning-based borrowing constraints.

With the identified structural shocks we construct new indicators for identifying financial frictions. Our preferred indicators are based on (i) directly thresholding the contemporeneous shock to financial frictions and (ii) thresholding the historical decomposition of debt and the marginal productivity of inputs in terms of the financial friction shocks.

To operationalize our identification approach we embed our linearized structural model in a flexible firm level structural panel vector autoregressive model. The reduced form parameters of this model can be consistently estimated using the panel GMM methods of

Cao and Sun (2011). To conduct inference on the structural elements of interest, e.g. the structural shocks, impulse responses or financial friction indicators, we follow the econometric literature on sign-based inference, and show how these methods can be adopted to estimate and conduct inference on the identified sets for the structural elements (e.g. Gafarov et al. 2018, Granziera et al. 2018).

Empirical validation The second part of the paper verifies the validity of our methodology for several datasets. First, we embed the baseline structural model into a realistically calibrated industry equilibrium framework, and we use it to simulate a panel of firm-level balance sheet data. We show that the structural shocks identified with our methodology are good approximation of the true shocks, and that our financial frictions indicator is very accurate in identifying financially constrained firms.

For real world data it is not easy to verify whether our financial frictions indicator is accurate, because of the lack of a reliable alternative measure. With this in mind, we first consider a panel of US Compustat firms, for which narrative-based measures of financial frictions are available, and we propose two tests. The first one is a quasi-natural experiment based on the 2008-2009 financial crisis. We measure the innovation to financial frictions for 2007 and argue that large values should negatively affect employment in 2008, at the beginning of the financial crisis, more so when compared to the effect of financial frictions on employment decisions prior to the financial crisis. We implement this test using the Compustat sample of manufacturing firms and find strong support for our hypothesis. The second test is the comparison of our indicator to the one constructed using the narrative method of Hoberg and Maksimovic (2014). We show that the two indicators are consistent with each other.

We then apply our indicator to the quasi universe of Spanish manufacturing firms above 5 employees. This is important because while the analysis on Compustat is useful for comparison purposes, it is a sample of large public companies, and we want to test our methodology on a more comprehensive dataset that includes also smaller firms, which are much more likely than larger ones to face significant financial frictions. After confirming on this dataset the results of the test based on the 2008-2009 financial crisis, we test two hypotheses. First, our estimated financial frictions shock should explain a larger part of employment variability for small than for large firms. Intuitively, smaller firms are more likely to be subject to credit shocks because of a "flight to quality" effect. When a lender is constrained in its funding, it will prioritise lending to its larger customers, insulating them from fluctuations in the availability of credit, and leaving the smaller customers more exposed. Second, we expect financial shocks to also be more persistent for small than for large firms. The intuition for the second hypothesis is that large firms not only have better access to their main lender, but also relations with multiple banks. If in a given period the firm is forced to reduce its employment level because of unexpected problems in accessing external financing, it is expected that it will be able to find alternative sources relatively quickly. Conversely, many smaller firms typically rely only on one main lender. If they face an increase in financing problems, they will be much less likely to find quickly suitable alternatives. Our empirical evidence strongly supports both hypotheses.

Relation to literature This paper broadly relates to two strands of literature: the identification and quantification of firm-level financial frictions, and the sign-based identification literature in macroeconomics and econometrics.

With respect to the financial frictions literature, we are motivated by the lack of reliable measures based on balance sheet data (e.g. Farre-Mensa and Ljungqvist 2015). Our proposed methodology is related to the large literature that uses the insights from dynamic corporate finance theory to identify financial frictions. In particular, since we focus on the optimality conditions of a dynamic model of firms' financing and investment decisions, our paper is related to earlier approaches based on the estimation of Euler equations for fixed capital investment (e.g., among others, Whited 1992, Hubbard et al. 1995, Love 2003, Whited and Wu 2006).[5] As we show in the context of the models analysed in this paper, a limitation of this approach is that in general the mapping between the latent intensity of financial frictions and observables is model specific, and hence not suitable to detect different types of financing imperfections.

Our approach differs in two main respects. First, we use multiple equations simultaneously to identify financial frictions. We show that, because these frictions have different effects on real and financial variables, using different equations leads to sufficient identifying restrictions to disentangle financial frictions shocks from the other shocks. Second, we use an identification strategy that overcomes the above-mentioned limitation, being consistent with several different models and types of financial frictions.[6]

Our paper is also related to the empirical literature that uses quasi natural experiments to identify the causal effect of financial frictions. Among others, see for example Chaney

[5]In related theory-based approaches, Caggese (2007) uses the Euler Equation for variable capital investment, while Cherchye et al. (2020) measure financial frictions as the difference between the firm's profitability with its actual inputs and the firm's achievable profitability with optimal inputs.

[6]More generally, our theory-based approach is related to the literature that develops and estimates firm investment models with financial frictions, as in Hennessy and Whited (2007). More recently, Nikolov et al. (2021) use the empirical policy function estimation technique introduced by Bazdresch et al. (2017), and Catherine et al. (2022) use a structural model to quantify the aggregate effects of financing constraints using well identified reduced form evidence on collateral constraints. These papers estimate the structural parameters of these models, and therefore provide useful information on the nature of financial frictions and their average intensity in the industry, but do not provide an identification strategy to identify time varying financial frictions at the firm-level, which is the objective of this paper.

et al. (2012), who use variations in local house prices and show that firms exposed to an exogenous increase in the collateral value of their assets increase their investment relative to the other firms. Another example is Chodorow-Reich (2014), who exploits the fact that bank-firm relations are sticky, and that the 2008-2009 financial crisis affected asymmetrically lenders depending on their exposure to the subprime market. He shows that firms borrowing from these exposed lenders reduced employment more than the other firms. Our objective is fundamentally different from these papers. We provide a novel-theory based methodology to measure the intensity of financial shocks at the firm level using only commonly available balance sheet data and without the need of an external shock such as the 2008 banking crisis or exogenous variations in property prices. Furthermore, we show that our estimated shocks can be used to construct reliable indicators to select financially constrained firms.

Moreover, our emphasis is on developing a financial constraints indicator that is consistent with different types of financial frictions, which is empirically important as shown in Lian and Ma (2021), Kermani and Ma (2020), Caglio et al. (2021) and Drechsel (2022), among others.

The sign restriction based identification approach that we use stems from the pioneering work of Faust (1998) and Uhlig (2005), and has become a popular identification strategy for aggregate time series applications, see Fry and Pagan (2011) for a review. By now a large number of papers have worked out robust inference methods for structural vector autoregressive models that are identified using sign restrictions (e.g. Baumeister and Hamilton 2015, Gafarov et al. 2018, Granziera et al. 2018, Arias et al. 2018, Giacomini and Kitagawa 2020). We adopt a frequentist approach, where in the first step the reduced form panel VAR coefficients are estimated using a panel GMM approach (e.g. Arellano and Bond 1991, Holtz-Eakin et al. 1988, Cao and Sun 2011) and in the second step the sets of structural impulse responses are recovered using optimization methods (e.g. Gafarov et al. 2018, Granziera et al. 2018). Robust confidence intervals, that have correct frequentist coverage, are obtained by similar optimization routines, then also taking into account the estimation uncertainty from the reduced form estimates. We refer to Canova (2007) and Canova and Ciccarelli (2013) for a more elaborate discussion regarding panel VAR models.

The remainder of this paper is organized as follows. The next section describes a simple but generic model from which we derive our sign restrictions that allow to identify the real and financial shocks. Section 3 provides the estimation methodology. The empirical exercises are presented in Section 4. Any references to sections, equations, lemmas etc. which start with “S” refer to the supplementary material.

2 Firm optimization and identification

In this section we describe a benchmark dynamic model of a firm subject to financial frictions. The model is used to derive restrictions for identifying firm specific shocks which are subsequently used for constructing indicators for identifying financially constrained firms. Further, we discuss how our identification strategy is consistent with several extensions of the model that allow for alternative ways to model financial frictions.

2.1 Baseline model

The firm produces a consumption good, with price normalized to one, using the decreasing returns to scale production function

$$y_t = z_t l_t^{\alpha} \ , \tag{1}$$

where y_t denotes output, l_t is the production input, z_t is an exogenous productivity process and the constant α satisfies $0 < \alpha < 1$. The production input is non-durable and not subject to adjustment costs.[7]

The production input l_t summarizes variable inputs that need to be paid at least partially in advance and are potentially subject to financial frictions. Prominent examples include labour and other variable production inputs such as materials. For convenience, we refer to l_t as labour and the price of one unit of l_t is the wage w. In this model, we assume that l_t needs to be financed in advance because labour hired in one period generates revenues at the beginning of the next period. At the beginning of period t the firm has financial resources equal to

$$s_t = y_{t-1} - b_{t-1} \ ,$$

where b_{t-1} is the face value of one-period debt borrowed in the previous period. The firm observes the new values of productivity z_t, financial frictions ξ_t, and θ_t, the latter being a process related to overhead costs of production. The latent stochastic processes $\log z_t$, $\log \xi_t$ and $\log \theta_t$ are allowed to be persistent. We denote their innovations by ε_t^z, ε_t^ξ and ε_t^θ which we refer to as productivity, financial frictions, and liquidity shocks, respectively. The budget constraint is given by

$$\mathrm{div}_t = s_t - \theta_t F - w l_t + \Theta(b_t, \xi_t) \ , \qquad \text{with} \qquad \Theta(b_t, \xi_t) \equiv \frac{b_t}{1+r} - c_t \ , \tag{2}$$

where div_t denotes dividends, and $\theta_t F$ are overhead costs of production, that vary over time with θ_t, while F is a positive constant. Since the purpose of θ_t is to cause fluctuations in

[7] While we think about the innovations to $\log z_t$ as productivity shocks, in the context of this model an equally plausible interpretation is that they capture demand shocks. Moreover, below we consider an extension of the model with additional inputs that are durable and subject to adjustment costs.

profits that generate additional liquidity needs, we denote the shock ε_t^θ as liquidity shock.[8] $\Theta(b_t, \xi_t)$ captures the funds obtained in period t from borrowing a face value of debt b_t to be repaid the next period, and c_t is the excess cost of external finance as described below. We assume that the firm is subject to financial frictions. First, the firm cannot issue equity while in operation, i.e.

$$\text{div}_t \geq 0 \ . \tag{3}$$

Second, we assume that b_t is subject to financial frictions that increase its cost. In the literature, researchers have considered a variety of asymmetric information or enforceability problems that limit the amount of debt firms can borrow. These different approaches imply limits to the access to external finance that can be classified in two broad categories: (i) frictions that determine a maximum borrowing limit and (ii) frictions that determine a premium in the risk-adjusted cost of credit, increasing in debt or leverage. In both cases, these frictions imply levels of borrowing and investment lower than it would be optimal in the absence of financial frictions. In the benchmark model of this section, we only consider the second approach and introduce a reduced-form excess cost cost of credit as a function of the amount borrowed. Importantly, in Section 2.3, we show that our results are confirmed in richer models that include also a borrowing limit and the presence of both asset-based and earnings-based collateral.

Specifically, here we assume that borrowing implies the additional cost

$$c_t = \begin{cases} \xi_t b_t^\gamma & \text{if} \quad b_t > 0 \\ 0 & \text{if} \quad b_t \leq 0 \end{cases} , \tag{4}$$

where $\gamma > 1$. Since in this model there is no capital and firms are ex ante identical, this formulation is equivalent to assuming that c_t is increasing and convex in leverage. In Section S2.1 in the Appendix we provide a microfoundation of this assumption. To illustrate, suppose that b_t is banking debt, and the bank expects that higher debt increases the probability that the firm refuses ex post to repay it, because it prefers an outside option to default on the debt and steal some of the assets purchased with it. The additional fee c_t ensures that the bank receives the required expected return from the loan.

Furthermore, notice that (4) potentially implies a kink in the cost of credit around $b_t = 0$. However, empirically we nearly exclusively observe positive values of debt, because while financial frictions reduce borrowing, other important factors such as the debt tax shield, or debt as an incentive device for managers, increase it. As shown below, in the model we include this feature by assuming the firm has a relatively high discount factor, so that has a

[8]Empirically these types of costs are significant. Arellano et al. (2019) show, in a model similar to the one considered in this paper, that shocks to fixed production costs are key to generate a dispersion in financial spreads as large as the ones observed in the data.

preference for distributing dividends and is a net borrower in the steady state. Therefore in the following analysis we focus on the identification of the shocks conditional on observing positive debt.

We interpret ξ_t in (4) as a latent process driving the excess cost of external finance, and its innovation ε_t^{ξ} as a financial frictions shock. An increase in ξ_t could be for example interpreted as a credit crunch that reduces a lender's ability to obtain funding and hence increases the return it requires on its loans. Notice the difference between ξ_t, which is an exogenous stochastic process, and c_t, which is an endogenous quantity that depends on the firm's decisions.

The objective of the firm is to maximise its intertemporal value $V_t(S_t)$, which is defined as the net present value of dividends. S_t is the vector of state variables.[9] We have:

$$V_t(S_t) = \max_{l_t, b_t} (1+\phi_t)\,\mathrm{div}_t + \frac{1}{1+r}\frac{1}{\mu}\mathbb{E}_t\left[V_{t+1}\left(S_{t+1}\right)\right] \ , \tag{5}$$

where ϕ_t is the Lagrange multiplier associated to the no dividends constraint (3). Therefore, $1+\phi_t$ represents the shadow value of having one additional unit of wealth in period t.[10] The presence of financial frictions implies firms have a preference for not distributing dividends and accumulating savings, which would make all firms net savers in equilibrium. To counteract this, in the literature it is often assumed either the presence of tax distortions that reduce the cost of debt, or that firms have a preference for distributing than retaining dividends, because their discount rate is higher than the interest rate. We take the latter option and assume that firms discount dividends at the rate $\frac{1}{1+r}\frac{1}{\mu}$, where $\mu > 1$.[11]

Assumptions

To separately identify the productivity, financial friction and liquidity shocks from observed output, debt and productivity series alone we impose the following basic assumptions.

[9] If the stochastic processes are AR(1), then $S_t = (s_t, \theta_t, z_t, \xi_t)$.

[10] Note that the maximization problem (5) implies that the firm is infinitely lived. That is, we implicitly restrict parameter values to a subset such that: i) The firm always finds it optimal to continue, because the net present value of its future profits is always positive in all states. ii) The firm is never forced to exit because is unable to borrow to repay its maturing debt. This restriction simplifies the analysis and derivations, but it is not essential for our results. In Appendix S2 we illustrate a slightly modified version of the model which includes endogenous exit (either voluntary or because of bankruptcy) and we use it to simulate the artificial panel data analysed in Section 4.1, which confirm all of our results.

[11] An alternative way to limit the accumulation of savings is to assume that constrained agents have finite lives, as in Bernanke et al. (1999) and Gertler and Kiyotaki (2015). Our assumption of impatient entrepreneurs follows Kiyotaki and Moore (1997). Alternatively, tax benefits of debt are introduced for example in Jermann and Quadrini (2012), while agency frictions between managers and shareholders are assumed in Arellano et al. (2019). These alternative ways to generate firm borrowing in the steady state would not change the main results of our analysis.

Assumption 1. *Decreasing marginal returns in the inputs. There exist constants* $\underline{\alpha}$ *and* $\overline{\alpha}$*, such that* $0 < \underline{\alpha} \leq \alpha \leq \overline{\alpha} < 1$.

Assumption 2. *Marginal excess cost of debt* c_t *is increasing in leverage:* $\gamma > 1$.

Assumption 1 continues to hold if we assume more than one input and constant returns to scale technology in all inputs, as long as at least one input is predetermined (see Section 2.3). The lower $\underline{\alpha}$ and upper $\overline{\alpha}$ bounds can be application specific.[12]

Assumption 2 is consistent with different types of financial imperfections usually considered in financial frictions models (see Section S2.1 for a microfoundation). Moreover, in our model extensions in Section 2.3, we relax it allowing for the presence of collateralized credit.

2.2 Identifying restrictions

Having described our baseline model we now show how its key economic mechanisms, i.e. decreasing returns to scale and increasing marginal costs of debt in leverage, can be exploited to identify the productivity, liquidity and financial friction shocks. To show this we proceed in three steps: (i) we derive the first order conditions for the firm, (ii) log linearize these, and then (iii) we show that the economic mechanisms yield sufficient sign and magnitude restrictions to partially identify the structural parameters of the log linearized model, which in turn allows to recover measures for the structural shocks without committing to the specific functional form assumptions of the model.

First order conditions

The first order condition for debt b_{t+1} is given by

$$(1+\phi_t)\left(\frac{1}{1+r} - \gamma\xi_t b_t^{\gamma-1}\right) + \frac{1}{1+r}\frac{1}{\mu}\left[\mathbb{E}_t\left(\frac{\partial V_{t+1}}{\partial s_{t+1}}\right)\frac{\partial s_{t+1}}{\partial b_t}\right] = 0 \ ,$$

where $\frac{\partial s_{t+1}}{\partial b_t} = -1$ and, for the envelope theorem, $\mathbb{E}_t(\frac{\partial V_{t+1}}{\partial s_{t+1}}) = \mathbb{E}_t(1+\phi_{t+1})$. It follows that

$$\frac{1}{1+r} = \psi_t\left(\frac{1}{1+r} - \gamma\xi_t b_t^{\gamma-1}\right) \ , \tag{6}$$

and

$$\psi_t \equiv \mu\frac{1+\phi_t}{\mathbb{E}_t(1+\phi_{t+1})} \ . \tag{7}$$

[12] For example, in our main empirical application in Section 4 which considers labour as the variable input, α is the elasticity of output to labour, and is a well known quantity, remarkably stable both over time and across countries, and often found in the range between $\underline{\alpha} = 0.4$ and $\overline{\alpha} = 0.8$.

Where ψ_t is the shadow value of finance in the firm today relative to the next period. The right hand side of (6) is the net present value of borrowing one additional unit of debt due in period $t+1$. The left hand side is the net present value of funds saved to obtain 1 unit of wealth in period $t+1$. The productivity of the firm determines the shadow cost ϕ_t of the non-negativity constraint on dividends. When productivity increases, such cost becomes higher relative to its expected value next period, thus increasing also ψ_t, and (6) is satisfied with equality by an increase in borrowing b_t. The first order condition for labour is

$$\frac{1}{1+r}\mathbb{E}_t\left(\frac{\partial V_{t+1}}{\partial s_{t+1}}\frac{\partial s_{t+1}}{\partial l_t}\right) - (1+\phi_t)w = 0 \ , \tag{8}$$

where $\frac{\partial s_{t+1}}{\partial l_t} = \alpha z_t l_t^{\alpha-1}$. Substituting we get

$$\frac{\alpha z_t}{l_t^{1-\alpha}} = (1+r)\,\psi_t w \ . \tag{9}$$

Intuitively, a firm with little wealth today finds it expensive to borrow, and chooses a sub-optimally low value of labour input l_t, which is reflected in high marginal product $\frac{\alpha z_t}{l_t^{1-\alpha}}$ and an high value of ψ_t from equation (9).

Linearized model

Next, we log linearize the first order conditions around the steady state with respect to debt and labour and combine these with the output equation. Specifically, we solve (6) with respect to b_t and (9) with respect to l_t to obtain

$$\begin{aligned} \log b_t &= c_b + \frac{\psi}{(\gamma-1)(\psi-1)}\log\psi_t - \frac{1}{\gamma-1}\log\xi_t \\ \log l_t &= c_l + \frac{1}{1-\alpha}\log z_t - \frac{1}{1-\alpha}\log\psi_t \\ \log y_t &= \log z_t + \alpha\log l_t \end{aligned} \tag{10}$$

where c_b and c_l are constants and ψ is the steady state value of ψ_t. From (7) it is possible to see that $\psi = \mu > 1$. The last equation comes from log linearizing (1).

The system (10) depends on the endogenous latent intensity of frictions ψ_t, which positively affects $\log b_t$. Intuitively, the tighter are financial frictions today relative to tomorrow, the higher is the shadow value of resources, the more the firm wants to borrow. The same intuition explains why l_t is negatively related to ψ_t. As it is clear from the previous discussion, ψ_t is an endogenous object. We claim, and later prove in Section S1, that in the log linearized model $\log \psi_t$ is equal to

$$\log \psi_t = c_\psi - \pi_1 \log s_t + \pi_2 \log \xi_t + \pi_3 \log \theta_t + \pi_4 \log z_t \ , \tag{11}$$

where $\pi_1, \ldots, \pi_4$ are positive coefficients. Substituting (11) into (10) and rearranging provides a linear mapping from the stochastic processes $g_t = \log(\xi_t, \theta_t, z_t)'$ to the observable variables $Y_t = \log(b_t, l_t, y_t)'$. Specifically we have

$$Y_t = c + D_s \log s_t + Bg_t \ , \tag{12}$$

where c is a vector of constants and

$$D_s = \pi_1 \begin{bmatrix} -\frac{1}{\gamma-1}\frac{\psi}{\psi-1} \\ \frac{1}{1-\alpha} \\ \frac{\alpha}{1-\alpha} \end{bmatrix} \ , \quad B = \begin{bmatrix} -\left(\frac{1}{\gamma-1} - \frac{\psi\pi_2}{(\gamma-1)(\psi-1)}\right) & \frac{\psi\pi_3}{(\gamma-1)(\psi-1)} & \frac{\psi\pi_4}{(\gamma-1)(\psi-1)} \\ -\frac{\pi_2}{1-\alpha} & -\frac{\pi_3}{1-\alpha} & \frac{1-\pi_4}{1-\alpha} \\ -\frac{\alpha\pi_2}{1-\alpha} & -\frac{\alpha\pi_3}{1-\alpha} & \frac{1-\alpha\pi_4}{1-\alpha} \end{bmatrix} . \tag{13}$$

The linearized structural model (12) relates the firm output and production variables to the firms financial resources s_t and the different stochastic processes. In practice, the latent variables g_t are correlated over time. To capture this we write

$$\tilde{A}(L) g_t = \varepsilon_t \ , \qquad \text{with} \qquad \tilde{A}(L) = I - \tilde{A}_1 L - \ldots - \tilde{A}_p L^p \ , \tag{14}$$

where L is the lag operator and $\varepsilon_t = (\varepsilon_t^\xi, \varepsilon_t^\theta, \varepsilon_t^z)'$ captures the serially uncorrelated innovations to the latent financial frictions, liquidity and productivity processes. We assume that the polynomial $\tilde{A}(L)$ is invertible.[13]

Substituting g_t into (12) and reordering the terms gives

$$Y_t = c + DW_t + A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + B\varepsilon_t \ , \tag{15}$$

where the coefficients satisfy $A_j = B\tilde{A}_j B^{-1}$, for $j = 1, \ldots, p$, $W_t = (\log s_t, \ldots, \log s_{t-p})'$ and $D = (D_s, A_1 D_s, \ldots, A_p D_s)$. The model (15) can be recognized as a structural vector autoregressive model for a given firm with predetermined explanatory variables W_t. Without loss of generality we assume that the structural shocks are normalized to be mutually uncorrelated and have mean zero and unit variance: $\mathbb{E}(\varepsilon_t) = 0$ and $\mathbb{V}(\varepsilon_t) = I_K$.

Sign and magnitude restrictions

The second moments of the observable variables in the linear system (15) only allow to recover the structural shocks up to orthogonal transformations (e.g. Kilian and Lütkepohl

[13] Formally, $\tilde{A}(z)$ satisfies $\det|\Theta(z)| \neq 0$ for all $z \in \mathbb{C}$ such that $|z| \leq 1$.

2017). In contrast, if the matrix B is parameterized as in (13), all parameters in B and the structural shocks can be recovered from the second moments of the observables.[14]

We can view these two scenarios as two extremes: on the one hand it seems excessive to assume that the model is completely uninformative about B, yet at the same time the specific parameterization of B in (13) is undoubtedly due to some of the specific modeling choices made for the baseline model. Indeed, below we show that reasonable changes in the baseline model lead to different parametrizations for B.

To this extent, the approach in this paper for identifying B and the structural shocks takes a middle ground between these two extremes. Specifically, we only use the parametrization of B in (13) to recover a set of sign and magnitude restrictions that allow us to set-identify B. The key benefit of this approach is that, as shown below in Section 2.3, these sign and magnitude restrictions hold under a wide set of alternative models, implying that even if the baseline model is misspecified (as it likely is in practice) the sign and magnitude restrictions are plausible.

Proposition 1. *For the baseline model defined in Section 2.1 we have that under Assumptions 1-2 the matrix B in* (13) *satisfies the following sign restrictions*

$$B = \begin{bmatrix} - & + & + \\ - & - & + \\ - & - & + \end{bmatrix} . \tag{16}$$

In addition, the following magnitude restrictions hold

$$\underline{\alpha} \leq \left\{ \frac{B_{31}}{B_{21}}, \frac{B_{32}}{B_{22}} \right\} \leq \overline{\alpha} \qquad \textit{and} \qquad \frac{B_{33}}{B_{23}} > 1 \ . \tag{17}$$

The proof is given in Appendix S1.

The economic intuition for the sign restrictions in (16) is as follows. A positive financial shock ε_t^{ξ} raises the cost of external finance ξ_t. The firm reduces debt b_t and therefore is also forced to reduce labour input l_t, which implies lower output y_t. A positive productivity shock ε_t^{z} increases the marginal return on labour and incentivises the firm to increase borrowing b_t to increase l_t. y_t also increases, both because of the direct effect of the productivity shock (more output y_t for given labour input) and because of the indirect effect of increasing l_t. A liquidity shock (that is, a positive value of ε_t^{θ}) increases θ_t and increases borrowing b_t. Moreover, by tightening financial frictions and increasing the marginal cost of debt, it implies the firm desires to hire less labour l_t and produces less output.

[14]This is easy to verify as in (13) there are five free parameters that determine the nine elements of B. These five parameters can all be recovered from the variance of the process $\{B\varepsilon_t\}$.

Notice that the financial shock ε_t^ξ and the productivity shock ε_t^z cannot be disentangled using the sign restrictions alone, because both shocks affect the 3 variables in the same direction. However, they can be separately identified thanks to the magnitude restrictions in (17). The economic intuition for the first magnitude restriction in (17) is that both a positive ε_t^θ shock and a positive ε_t^ξ shock, all else equal, increase financial frictions. Since the marginal productivity of labour is decreasing, when the firm is forced to reduce the labour input because of stronger financial frictions, then labour productivity increases and the fall in output is mitigated relative to the fall in labour.[15] Regarding the second inequality in (17), the economic intuition of why the elasticity of output to the productivity shock (relative to the elasticity of labour) is larger than for the other shocks is also simple. An increase in z_t increases output $\log y_t$ both directly in the production function and indirectly because it also increases labour input. Conversely the other shocks only affect output indirectly through their effect on labour input.

2.3 Robustness to alternative model specifications

As the general discussion surrounding the identifying assumptions eluded to, the derived sign and magnitude restrictions rely on simple economic mechanisms, which are likely to hold in many other dynamic models with possibly different types of financial frictions. Here we show that this is indeed the case: the restrictions implied by the baseline model are robust to several model extensions and different ways of micro-founding the financial frictions. We keep the exposition brief, merely stating the types of deviations for which our results hold, but in Appendix S2 we provide detailed derivations and the formal results that effectively extend Proposition 1.

2.3.1 Capital in the production function

We first extend the baseline model by adding capital in the production function

$$y_t = z_t l_t^\alpha k_t^\beta \ . \tag{18}$$

With $\beta > 0$ and $0 < \alpha + \beta \leq 1$. We assume that capital takes one period to install, and therefore k_t is pre-determined at time $t-1$.[16]

The optimization problem becomes:

[15] Models that include customer capital and/or inventories are also consistent with this prediction. Financially constrained firms are known to maximise current revenues while controlling input costs, either by running down inventories, or by increasing prices.(e.g. Gilchrist et al. 2017)

[16] To be precise, capital takes one period to install and two periods to generate revenues. At the beginning of period t, the firm has installed capital k_t, and chooses l_t and k_{t+1}. Labour l_t generates revenues at the end of period t, while capital k_{t+1} generates revenues at the end of period $t+1$.

$$V_t(S_t, k_t) = \max_{l_t, b_t, k_{t+1}} (1+\phi_t)\text{div}_t + \frac{1}{1+r} E_t\left[V_{t+1}(S_{t+1}, k_{t+1})\right], \tag{19}$$

subject to the budget constraint:

$$\Theta(b_t, \xi_t) = -s_t + \theta_t F + w l_t + \text{div}_t + f(i_t), \tag{20}$$

where i_t is net investment,

$$i_t \equiv k_{t+1} - (1-\delta)k_t,$$

and $0 < \delta < 1$ is the depreciation rate of capital. The function $f(i_t)$ represents the total expenditure to invest, which includes the direct cost i_t as well as capital adjustment costs. Since we do not plan to use the first order condition for k_{t+1} to identify the shocks, we do not need to fully specify capital adjustment costs and the investment policy function. More specifically, we only impose that, conditional on current wealth s_t and installed capital k_t, productivity z_t affects investment i_t directly, while the latent variables ξ_t and θ_t affect it only indirectly, by changing the latent shadow cost ψ_t, same as it happens for labour input (see the system 10). Hence $f(i_t)$ can be represented as a function of s_t, k_t, z_t and ψ_t, and log-linearising yields:

$$\log f(i_t) = \epsilon_s^i \log s_t - \epsilon_\psi^i \log \psi_t + \epsilon_z^i \log z_t + \epsilon_k^i \log k_t, \tag{21}$$

where we do not need to impose any restriction on ϵ_s^i and ϵ_k^i. Regarding the elasticities with respect to the productivity shock z_t and to ψ_t, we only impose that investment weakly decreases in financial frictions, and weakly increases in productivity. hence:

$$\epsilon_\psi^i \leq 0; \epsilon_z^i \geq 0 \tag{22}$$

These restrictions are very mild and consistent with all the types of capital adjustment costs commonly considered in the literature, such as convex adjustment costs, fixed adjustment costs, and irreversibility. In particular, we allow ϵ_ψ^i and ϵ_z^i to be equal to zero to be consistent with the potential presence of non-convex adjustment costs which create inaction regions in the optimal investment policy. For example, if fixed capital is irreversible, ϵ_ψ^i might be equal to zero when, after an increase in financial frictions which reduces funds available to purchase capital and labour, the irreversibility constraint binds. Likewise, ϵ_z^i might be also equal to zero when the firm has an inefficiently high level of installed capital k_t. In Appendix S2 we derive and log-linearize the first order conditions for b_t and l_t and show that this model yields the same system (12) and set of identifying restrictions (16) and (17) of the benchmark model, with the only difference that the vector of predetermined variables is now $W_t = (\log s_t, \log k_t)'$ instead of $\log s_t$.

2.3.2 Models with collateralized borrowing

We consider extensions of the baseline model that allow for collateralized credit, where collateral can be both assets and revenues. While a large literature in macro-finance has emphasised constraints of the first type, a recent literature points out the larger empirical relevance of the second type (e.g. Lian and Ma 2021). For clarity of exposition we consider the two forms of borrowing separately, even though to consider them jointly would not affect the results.

Importantly, we not only show that our identification restrictions are valid in these alternative models, but also that they are able to capture financial frictions shocks that affect both the price of credit (the shock to the cost of credit ξ_t for a given loan to value ratio) and its quantity (a shock to the loan to value ratio).

Asset based borrowing

We consider the model with capital analysed in Section 2.3.1 and modify equation (4) as follows

$$c_t = \begin{cases} \xi_t \left(\frac{b_t}{(1-\lambda_t^1)(1-\delta)k_t} \right)^{\gamma} & \text{if} \quad b_t > (1-\lambda_t^1)(1-\delta)k_t \\ 0 & \text{if} \quad b_t \leq (1-\lambda_t^1)(1-\delta)k_t \end{cases}, \tag{23}$$

where $(1-\lambda_t^1)(1-\delta)k_t$ is the collateral value of the firm's current physical assets and $0 < \lambda_t^1 < 1$ is the fraction of value of capital that *cannot* be used as collateral. One way to micro-found this assumption is to assume that the lender, in case the debt is not repaid, is able to liquidate the firm and the residual value of current assets is $(1-\lambda_t^1)(1-\delta)k_t$.[17] In other words, the firm is able to borrow at the risk free rate as long as debt is below the value of the firm's collateral. Any borrowing above such level incurs in additional costs proportional to the debt to collateral assets ratio.[18]

Notice that in this formulation a shock that increases ξ_t has the same interpretation as in the benchmark model, is a shock that increases the cost of credit. Conversely, a shock that increases λ_t^1 reduces the collateral value of capital, and hence the availability of low cost borrowing. The firm maximises the value function (19) subject to (20), (21), (22) and (23). In Appendix S2 we show that this model yields the same system and set of identifying

[17] Therefore, current capital remains in place until debt is repaid at the beginning of period $t+1$. Conditional on repayment of the debt, the new capital is installed. In case of no repayment, the firm is liquidated.

[18] In the benchmark model we were focusing on firms with positive values of debt, so that $c_t > 0$. Likewise, for the models in this section, we focus on firms that have positive uncollateralised debt. In the data, firms that borrow collateralised debt usually also have positive uncollateralised borrowing such as short term banking debt, lines of credit, or trade credit.

restrictions described above, also in this case with $W_t = (\log s_t, \log k_t)'$. In particular, we also show that the same set of sign and inequality restrictions apply whether we consider the shock to affect the price of credit (ξ_t) or the the quantity of credit (λ_t^1). Therefore, our estimated financial shock is able to capture both an increase in the price of credit as well as tighter quantity constraints.

Earnings based borrowing

We finally consider the possibility of earnings based borrowing. We go back to the benchmark model without capital of Section 2.1, and we follow Drechsel (2022) by assuming that borrowing capacity is proportional to profits. Therefore, we modify equation (4) as follows:

$$c_t = \begin{cases} \xi_t \left(\frac{b_t}{(1-\lambda_t^2)\pi_t} \right)^\gamma & \text{if} \quad b_t > \pi_t \\ 0 & \text{if} \quad b_t \leq \pi_t \end{cases} . \tag{24}$$

Where profits π_t are the net present value of revenues net of variable costs:

$$\pi_t \equiv \frac{y_t}{1+r} - wl_t \tag{25}$$

One way to micro-found this assumption is to assume that the lender, in case the debt is not repaid, is able to seize the fraction $1 - \lambda_t^2$ of firms' profits to repay the debt. Apart from equation (24), we follow here the benchmark model. The derivation of this model is more complicated because of the presence of a feedback effect between input demand and the collateral constraint. Notice that the first order condition (9) implies that the effective cost of labour is higher than the wage bill, because it includes the relative shadow cost of resouces ψ_t. Therefore optimal labour input is below the level that maximises profits, and with an earning-based borrowing constraint, there is an additional motive to expand the labour input to increase revenues and relax the constraint. The details of the derivations are in Appendix S2. We show that even though this constraint changes the magnitude of several reduced form coefficients, all our sign and inequality restrictions remain valid.[19]

2.4 Identifying financially constrained firms

Proposition 1 enables the partial identification of the contemporaneous effects of the structural shocks on debt, labour and output. Moreover, given a set for B we can identify other

[19]Notice that, in the case of earning based borrowing, a positive shock to z_t has potentially ambiguous effects on financial frictions because it implies additional demand for funds to invest, but also additional revenues generated that relax the borrowing constraint. In the Appendix, we show that the first effect dominates under very mild assumptions.

structural functions of interest. Specifically, since D and $A_1, \ldots, A_p$ are identifiable from the autocovariances of the process $\{Y_t, W_t\}$ we can set identify any function of B, D, $A_1, \ldots, A_p$ and the data.

For instance, this enables the recovery of the structural shocks $\varepsilon_t = (\varepsilon_t^\xi, \varepsilon_t^\theta, \varepsilon_t^z)'$, impulse responses and forecast error variance decompositions (e.g. Kilian and Lütkepohl 2017, Chapter 4). Here we will be mainly interested in constructing indicators for identifying financially constrained firms.

For this several authors have considered the Lagrange multiplier representing the relative shadow cost of external finance, i.e. ψ_t, as a theory-based indicator of financial frictions. Since this shadow cost is unobservable, previous literature has approximated it with a function of observable firm characteristics (e.g., among others, Whited 1992, Hubbard et al. 1995, Love 2003, Whited and Wu 2006). However, there is no guarantee that such approximations have general validity, and Farre-Mensa and Ljungqvist (2015) have shown that financial frictions indicators based on extrapolations of this approach have limited ability to identify financially constrained firms in practice. In our framework, we also have face a similar limitation. Equation (11) represents $\log \psi_t$ as a linear function of the three latent stochastic processes g_t. However, as explained in Appendix S2, the coefficients π_1-π_4 are model specific, and therefore we cannot obtain an estimate of ψ_t that has the same general validity of our estimated shocks ε_t.

Therefore, our preferred indicator is based directly on the financial friction shock ε_t^ξ. According to our identification strategy, a positive ε_t^ξ shock increases external financing costs, reduces borrowing, and reduces firm inputs below the optimal choice of the firm if it were financially unconstrained, so that the firm operates with an inefficiently high marginal product of labour. As such, firms with a larger value of this shock are likely more financially constrained than the other firms. Therefore, our benchmark indicator is a binary variable equal to one for the firms with the highest value of ε_t^ξ, and zero otherwise:

$$\mathcal{I}_t^\xi = \begin{cases} 1 & \text{if} \quad \varepsilon_t^\xi > \tau \\ 0 & \text{else} \end{cases} , \tag{26}$$

where τ is a user chosen threshold that is typically based on the distribution of the firm level shocks. For instance, if we believe that on average 10% of firms are financially constrained we can choose τ such that it corresponds to the cross-sectional quantile that implements this requirement. In defining an indicator that select firms in two groups of likely financially constrained and unconstrained firms, we follow the strategy normally adopted in the finance literature. Nonetheless, in our empirical analysis we also verify the robustness of the results in considering the shock ε_t^ξ itself as the indicator of financial frictions.

Furthermore, notice that ε_t^ξ measures the shock to financial frictions costs, rather than their level. This is an advantage rather than a limitation, because in our general framework it is not the *absolute* level of financial frictions that affect real decisions of firms, but rather their *relative* value.[20] This result is well known in this literature, as also highlighted by Gomes et al. (2006) and Whited and Wu (2006), who notice that "*finance constraints can only affect investment if they are time varying. It is the shadow value of the constraint today, relative to tomorrow, that is important*".

Therefore our indicator $\mathcal{I}_t^{\xi}$, since it captures firms with a large financial friction shock ε_t^{ξ}, is a good candidate to identify firms with an higher *relative* shadow cost of external finance than the other firms. In Appendix S4 we confirm that this is the case, using data simulated from a realistically calibrated version of our benchmark model.

We also consider alternative financial frictions indicators based on the historical decomposition of debt and labour productivity in terms of the financial frictions shocks.[21] Specifically, let $\lambda_{d,\xi,h}$ denote the horizon h impulse response of ε_t^{ξ} on debt. The part of debt that is due to financial frictions is then given by

$$\log b_t^{\xi} = \sum_{h=0}^{t-1} \lambda_{b,\xi,h} \varepsilon_{t-h}^{\xi} \ .$$

The advantage of using the historical decomposition is that it takes into account lagged financial friction shocks. Similar as above we can also threshold the historical decomposition to construct a binary indicator

$$\mathcal{I}_t^{b} = \begin{cases} 1 & \text{if} \quad \log b_t^{\xi} < \tau \\ 0 & \text{else} \end{cases} . \tag{27}$$

Firms identified with $\mathcal{I}_t^b = 1$ received a sequence of shocks that increased the cost of external finance and reduced their debt, relative to the other firms. We also perform the same decomposition on the other endogenous variables to obtain $\log y_t^{\xi} - \log l_t^{\xi}$, which is the part of average product of labour due to the financial frictions shock, and compute:

$$\mathcal{I}_t^{\frac{y}{l}} = \begin{cases} 1 & \text{if} \quad \log y_t^{\xi} - \log l_t^{\xi} > \tau \\ 0 & \text{else} \end{cases} . \tag{28}$$

While the $\mathcal{I}_t^b$ indicator focuses on the financial consequences of the shock (firms are forced to reduce their debt), the indicator $\mathcal{I}^{\frac{y_t}{l_t}}$ focuses on its real consequences. Firms identified with

[20]More precisely, the level of the shadow cost of external finance in period t is ϕ_t. That is, ϕ_t measures the increase in value in the firm if its financial assets increase by one unit. But any firm investment decision that has an inter-temporal nature is instead affected by ψ_t, which, as shown in equation (7), is the value of ϕ_t *relative* to its the expected value in period $t+1$.

[21]See Kilian and Lütkepohl (2017, page 115) for details on historical decompositions.

$\mathcal{I}^{\frac{y_t}{l_t}} = 1$ are those that reduce the most their labour input relative to their output because of the current and past financial frictions shocks.

In our empirical work below we use both our benchmark measure $\mathcal{I}_t^{\xi}$, as well as the alternative measures $\mathcal{I}_t^{b}$ and $\mathcal{I}^{\frac{y_t}{l_t}}$, to identify financially constrained firms.

3 Empirical methodology

In this section we show how the set of sign and magnitude restrictions in Proposition 1 can be exploited to identify and estimate real and financial shocks at the firm level, and construct indicators for financially constrained firms.

We assume that we observe outcome variables for N firms for a total of T time periods. The $K = 3$ observable variables —debt, labour and output— are summarized in the vector $Y_{i,t} = (\log b_{i,t}, \log l_{i,t}, \log y_{i,t})'$ where i indexes the firm and t the time period. Additionally, the $L \times 1$ vector $W_{i,t}$ denotes the predetermined variables, which may include lagged savings and capital. In the linearized baseline model (12) only savings are included, but the extensions from Section 2.3 and in the empirical applications, we also include capital in $W_{i,t}$.

As derived in (15) the firm level model can be written as

$$Y_{i,t} = c_i + d_t + DW_{i,t} + A_1 Y_{i,t-1} + \ldots + A_p Y_{i,t-p} + B\varepsilon_{i,t} \ , \tag{29}$$

where the only difference is that we include c_i —a firm level fixed effect— and d_t —a time fixed effect—. The fixed effects are intended to capture any time- or firm-invariant differences between the firms over time that are not explained in the model. We can view (29) as a K-dimensional Structural Panel Vector Autoregressive (SPVAR) model (e.g. Holtz-Eakin et al. 1988, Cao and Sun 2011).

The $K \times K$ coefficient matrix B is restricted by the sign and inequality restrictions derived in Proposition 1. Importantly, as explained in more detail below these restrictions are generally not sufficient to point identify B, but will merely shrink the identified set of admissible B matrices.

We proceed as follows. First, we briefly recall the estimation of the reduced form coefficients of the panel VAR following the approach of Cao and Sun (2011). Second, we show how to construct identified sets and confidence regions for different elements of the structural model. For the latter we adopt a frequentist approach based on projection arguments, see also Granziera et al. (2018).

3.1 Reduced model inference

The common reduced form parameters of model (29) are summarized in the vector μ which includes the regression coefficients D, the autoregressive coefficient matrices A_j and the variance matrix of the reduced form residuals denoted by Σ. We have that

$$\mu \equiv (\text{vec}(\Phi)', \text{vech}(\Sigma)')' \ , \qquad \Phi = (D, A_1, \ldots, A_p) \ , \qquad \Sigma = BB' \ . \tag{30}$$

We estimate the reduced form coefficients using the panel GMM approach discussed in Cao and Sun (2011). To illustrate this approach let $u_{i,t} = B\varepsilon_{i,t}$ denote the reduced form shocks.

We first remove the time fixed effects by subtracting the cross-sectional mean from each variables, i.e. we compute $Y_{i,t,j} - \frac{1}{N}\sum_{i=1}^{N} Y_{i,t,j}$ and $W_{i,t,k} - \frac{1}{N}\sum_{i=1}^{N} W_{i,t,k}$ for all i, j, k, t. Subsequently we take first differences of the demeaned variables to remove the firm-level fixed effects.

The resulting cross-sectionally demeaned SPVAR model in first differences is given by

$$\Delta Y_{i,t} = D\Delta W_{i,t} + A_1 \Delta Y_{i,t-1} + \ldots + A_p \Delta Y_{i,t-p} + \Delta u_{i,t} \ , \tag{31}$$

which effectively eliminates the fixed effects. The GMM estimator for μ relies on moment conditions that identify these parameters. Following Arellano and Bond (1991) a suitable set of moment conditions is given by

$$\mathbb{E}_F(\Delta u_{i,t} Y'_{i,t-1-l}) = 0 \ , \quad \mathbb{E}_F(\Delta u_{i,t} W'_{i,t-l}) = 0 \ , \quad l = 1, 2, \ldots, t+p-1 \quad t = 1, \ldots, T \ . \tag{32}$$

which requires the reduced form shocks to be serially uncorrelated. Exact assumptions are spelled out in Assumption S1 as stated in Appendix S3. To define the estimator for μ it is convenient to write the model in vector form. First, we define the first difference vectors

$$\underbrace{\Delta Y_i}_{T\times K} = \begin{bmatrix} \Delta Y'_{i,1} \\ \vdots \\ \Delta Y'_{i,T} \end{bmatrix} \quad \underbrace{\Delta u_i}_{T\times K} = \begin{bmatrix} \Delta u'_{i,1} \\ \vdots \\ \Delta u'_{i,T} \end{bmatrix} \quad \underbrace{\Delta X_{i,t}}_{(L+Kp)\times 1} = \begin{bmatrix} \Delta W_{i,t} \\ \Delta Y_{i,t-1} \\ \vdots \\ \Delta Y_{i,t-p} \end{bmatrix} \quad \underbrace{\Delta X_i}_{T\times(L+Kp)} = \begin{bmatrix} \Delta X'_{i,1} \\ \vdots \\ \Delta X'_{i,T} \end{bmatrix} .$$

These imply that the model for ΔY_i is given by

$$\Delta Y_i = (I_K \otimes \Delta X_i)\text{vec}(\Phi) + \Delta u_i \ .$$

The moment conditions (32) imply that the following instruments can be used for estimation.

$$Z_i = \begin{bmatrix} \tilde{Z}'_{i,1} & 0 & \dots & 0 \\ 0 & \tilde{Z}'_{i,2} & 0 & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \tilde{Z}'_{i,T} \end{bmatrix} = \begin{bmatrix} Z'_{i,1} \\ Z'_{i,1} \\ \vdots \\ Z'_{i,T} \end{bmatrix}$$

where $\tilde{Z}_{i,t} = (W'_{i,-p+1}, \dots W'_{i,t-1}, Y'_{i,-p}, \dots Y'_{i,t-2})'$. With these instruments the efficient GMM estimator for $\phi = \text{vec}(\Phi)$ is given by

$$\hat{\phi} = \text{vec}\left\{ \left[S'_{ZX} S^{-1}_{ZZ} S_{ZX} \right]^{-1} S'_{ZX} S^{-1}_{ZZ} S_{ZY} \right\} \tag{33}$$

where $S_{ZX} = \frac{1}{N}\sum_{i=1}^{N} Z'_i \Delta X_i$, $S_{ZY} = \frac{1}{N}\sum_{i=1}^{N} Z'_i \Delta Y_i$ and $S_{ZZ} = \frac{1}{N}\sum_{i=1}^{N} Z'_i G Z_i$, with G a tri-diagonal matrix with two on the main diagonal and minus one of the first sub-diagonals, see Cao and Sun (2011). The variance matrix Σ is estimated by

$$\widehat{\Sigma} = \frac{1}{NT}\sum_{t=1}^{T}\sum_{i=1}^{N} \hat{u}_{i,t}\hat{u}'_{i,t} \qquad \hat{u}_{i,t} = (y_{i,t} - y_{i,.}) - \widehat{\Phi}'(X_{i,t} - X_{i,.}) \tag{34}$$

where $y_{i,.}$ and $X_{i,.}$ denote the time averages.

Cao and Sun (2011) show that, under mild regularity conditions (see Theorem S1 in the appendix for a formal statement), the estimates $\hat{\phi}$ and $\text{vech}(\widehat{\Sigma})$ are $\sqrt{N}$-consistent and have a normal limiting distribution. Recalling the definition of the reduced form parameters in (30), we have that for $N \to \infty$ the reduced form parameters satisfy

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} N(0, \Omega) \ . \tag{35}$$

To save space we provide the details for this result in Appendix S3. There we also provide an estimator for the asymptotic variance that we denote by $\widehat{\Omega}$, which satisfies $\widehat{\Omega} \xrightarrow{p} \Omega$ as $N \to \infty$.

3.2 Structural model inference

Having described the estimation of the reduced form model parameters we discuss inference for several elements of the structural model. Specifically, we show how the sign and magnitude restrictions of Proposition 1 can be used to recover: (i) structural impulse responses and (ii) the structural shocks ε_t. By combining them we can estimate the indicators for financially constrained firms in (26)-(28).

Structural impulse responses

Since the parameters $A = (A_1, \dots, A_p)$ and B are common, the structural impulse responses are independent of i. In particular, for any given variable l, shock k and horizon h the corresponding structural impulse response is given by

$$\lambda_{l,k,h}(A, B) = e_l' C_h(A) B e_k \ ,$$

where e_l and e_k denote the l-th and k-th column of the identity matrix I_K and the $K \times K$ matrix $C_h(A)$ is defined recursively for $h = 0, \dots, H$ by

$$C_h(A) = \sum_{m=1}^{h} C_{h-m}(A) A_m$$

with $C_0(A) = I_K$ (e.g. Kilian and Lütkepohl 2017, Chapter 4). We note that the elements of $\lambda_{l,k,0}$ correspond to the elements of B.

The matrix B is such that $\Sigma = BB'$ and it satisfies the collection of sign and magnitude restrictions given in Proposition 1. We summarize these restrictions in $\mathcal{R}(\mu)$. These restrictions are not sufficient to uniquely recover B from μ. Instead for any μ there exists a set of matrices B that satisfy the restrictions. Formally, the identified set for the structural impulse responses $\lambda_{l,k,h}(A, B)$, for $h = 0, \dots, H$, is defined as

$$\mathcal{I}_{l,k}^{\mathcal{R}}(\mu) = \left\{ v \in \mathbb{R}^H : v_h = \lambda_{l,k,h}(A, B),\ \Sigma = BB',\ B \in \mathcal{R}(\mu) \right\} \ . \tag{36}$$

The upper and lower bounds of the elements of this set are defined by

$$\begin{aligned} \overline{v}(\mu) &= \sup\nolimits_{B \in \mathbb{R}^{K \times K}} \lambda_{l,k,h}(A, B) \quad s.t. \quad \Sigma = BB',\ B \in \mathcal{R}(\mu) \\ \underline{v}(\mu) &= \inf\nolimits_{B \in \mathbb{R}^{K \times K}} \lambda_{l,k,h}(A, B) \quad s.t. \quad \Sigma = BB',\ B \in \mathcal{R}(\mu) \end{aligned} \ . \tag{37}$$

Given estimates for A and Σ (as obtained in the previous section), the upper and lower bounds, for each l, k, h can be computed using numerical optimization methods (e.g. Gafarov et al. 2018). In practice, we use the SQP algorithm that is discussed in detail in Nocedal and Wright (2006). This gives an estimate for the identified sets of impulse responses.

Importantly, simply plugging in $\widehat{A}$ and $\widehat{\Sigma}$ only yields estimates for the upper and lower bounds of the set and does not take into account the estimation uncertainty of the reduced form parameters. To adjust the sets for estimation uncertainty note that the limiting distribution (35) implies the following estimate for the $1 - \alpha$ confidence region for the reduced form parameters.

$$CS_N(1-\alpha,\mu)=\left\{\mu\in\mathbb{R}^{d_\mu}:N(\hat{\mu}-\mu)'\widehat{\Omega}^{-1}(\hat{\mu}-\mu)\leq\chi^2_{1-\alpha,d_\mu}\right\} \tag{38}$$

where $\chi^2_{1-\alpha,d_\mu}$ denotes the $1-\alpha$ critical value of the χ-squared distribution with d_μ degrees of freedom, with d_μ the dimension of μ. Under the assumptions stated in Appendix S3 this Wald ellipsoid covers the true reduced form parameters with probability $1-\alpha$ as $N\to\infty$.

Using the confidence region for the reduced form parameters we can construct confidence intervals around the structural impulse responses. In particular, for the impulse response $\lambda_{l,k,h}$ we have

$$CS_N(1-\alpha,\lambda_{l,k,h})=\left[\inf_{\mu\in CS_N(1-\alpha,\mu)}\underline{\upsilon}(\mu),\sup_{\mu\in CS_N(1-\alpha,\mu)}\overline{\upsilon}(\mu)\right], \tag{39}$$

where $\underline{\upsilon}(\mu)$ and $\overline{\upsilon}(\mu)$ are defined in (37) as the lower and upper bounds of the identified set for a given vector of reduced form parameters μ. The confidence interval (39) is based on the "worst case" values of the reduced form parameters that lie within the confidence region of the structural shocks. The computation of these confidence intervals is again done by numerical methods, the difference with the *estimates* for the identified sets is that we now also optimize over the admissible set of reduced form parameters, instead of only over B. Gafarov et al. (2018) discuss several approaches for making the optimization feasible and we follow their approach by jointly optimizing over B and μ using the aforementioned SQP algorithm.

Based on these individual confidence intervals we may define the confidence region for the vector of impulse responses $\lambda_{l,k}=(\lambda_{l,k,1},\ldots,\lambda_{l,k,H})$, where H is the largest horizon considered, by

$$CS_N(1-\alpha,\lambda_{l,k})=CS_N(1-\alpha,\lambda_{l,k,1})\times\ldots\times CS_N(1-\alpha,\lambda_{l,k,H})\subseteq\mathbb{R}^H. \tag{40}$$

Similar as in Gafarov et al. (2018) this confidence region has correct frequentist coverage under mild assumptions. We note that this set is effectively based on a Bonferroni correction and as such it is conservative, see also Granziera et al. (2018) and Hoesch et al. (2022) who adopt similar methods to construct confidence sets in the context of SVAR models.

Structural shocks and indicators

Next, we discuss how the structural shocks can be recovered. Since, $u_{i,t}=B\varepsilon_{i,t}$, it follows that if B is known, or estimable, we may consider $\hat{\varepsilon}_{i,t}=B^{-1}\hat{u}_{i,t}$ and thus infer the structural shocks from the reduced form shocks as defined in (34). In our setting however there exists a range of matrices B that satisfy the sign and zero restrictions and thus there are multiple candidates for the structural shocks. In practice, when we wish to use the shocks for further analysis we rely on the median shock estimates and conduct robustness checks using the minimum and maximum values of the identified set, see also Baumeister and Hamilton (2019).

Given the structural shocks and the impulse responses we can estimate our indicators for identifying financially constrained firms. Specifically, the empirical counterparts of (26)-(28) are given by

$$\widehat{\mathcal{I}}^{\xi}_{i,t} = \begin{cases} 1 & \text{if} \quad \hat{\varepsilon}^{\xi}_{i,t} > \tau \\ 0 & \text{else} \end{cases} \tag{41}$$

$$\widehat{\mathcal{I}}^{b}_{i,t} = \begin{cases} 1 & \text{if} \quad \log \hat{b}^{\xi}_{i,t} > \tau \\ 0 & \text{else} \end{cases} \tag{42}$$

$$\widehat{\mathcal{I}}^{\frac{y}{l}}_{i,t} = \begin{cases} 1 & \text{if} \quad \log \hat{y}^{\xi}_{i,t} - \log \hat{l}^{\xi}_{i,t} > \tau \\ 0 & \text{else} \end{cases} \tag{43}$$

where $\log \hat{x}^{\xi}_{i,t} = \sum_{h=0}^{t-1} \lambda_{x,\xi,h} \hat{\varepsilon}^{\xi}_{i,t-h}$, with $x = b, l, y$, and the impulse responses $\lambda_{x,\xi,h}$ are typically taken as the median impulse responses of the estimated identified sets.

4 Empirical studies

In this section, after verifying the validity of our methodology on simulated data, we evaluate it on empirical firm level panel data. We focus on manufacturing firms to reduce cross industry heterogeneity. Nonetheless, the methodology could be also applied to other sectors. We first analyse a sample of US firms from Compustat. This is a useful benchmark to evaluate our methodology, because we can compare our results on the effects of financial frictions with those from other studies using the same data and period. Importantly, we can compare our financial frictions indicator with one based on narrative information. Then we apply our methodology to a large sample of firms from SABI, comprising the quasi universe of Spanish firms.

We consider an estimation procedure consistent with all the alternative models described in Section 2.3, which in practice implies adding the beginning of period stock of fixed capital as predetermined explanatory variable.

4.1 Simulated data

In the supplementary material section S4 we show the results from a simulation study. We use the structural model described in Section 2 to simulate an artificial industry, and draw a panel of $N = 10000$ firms for $T = 10$ periods. The parameters of the model are calibrated to match key moments related to the dynamics of productivity and financial variables at the firm level. In other words, we make sure that the simulated panel of firms is as much as possible realistic along the main dimensions of interest. With the simulated data we obtain three main findings: First, we show that the structural shocks identified with our methodology are good approximation of the true shocks. Second, we show that, in a realistically calibrated industry, our financial frictions indicator $\mathcal{I}_t^{\xi}$ defined in (26) is very accurate in identifying financially constrained firms. Third, these results apply regardless of whether we choose lower bound, median or upper bound structural shocks from our identified set. For a detailed illustration of these results, see Section S4.

4.2 Large US manufacturing firms

We consider manufacturing firms (SIC codes between 2000 and 3999), and we include all firm-year observations with at least 20 employees. We measure output $y_{i,t}$ with total revenues (Compustat variable *"sale"*), debt $b_{i,t}$ with the sum of short term (*"dlc"*) and long term debt (*"dltt"*), variable input $l_{i,t}$ with the cost of goods sold (*"cogs"*), fixed capital with property, plant and equipment (*"ppent"*), and financial wealth $a_{i,t}$ with cash and short term investments (*"che"*). We deflate these variables using the GDP deflator, and we winsorise the 1st and 99th percentiles of their distribution. Furthermore, we exclude outliers in the production technology by censoring the 1% tails of the distribution of ratio of cost of good sold to revenues.

We estimate the upper, median and lower bounds of the structural shocks $\hat{\varepsilon}_{i,t}$ using the following procedure. We consider the SPVAR model (29) with $p = 2$ lags and including savings and fixed capital as pre-determined variables. First, we estimate the reduced form model parameters using equations (33) and (34). Second, given the estimates we solve the programs (37) for $h = 0$ and all $l, k = 1, \ldots, K$, which corresponds to the recovery of B. We then recover the structural shocks $\hat{\varepsilon}_{i,t} = B^{-1}\hat{u}_{i,t}$ corrsponding to the median B as well as the upper and lower bounds for B. The restrictions that we impose are those derived in Proposition 1 with $\underline{\alpha} = 0.4$ and $\overline{\alpha} = 0.8$.[22] Then we determine whether firms are financially constrained. Regarding our benchmark indicator defined in Equation (41), we select our benchmark threshold τ such that we classify as constrained the firms with the 25% highest

[22] In practice this means imposing the very mild assumption that the output elasticity to labour input is between 0.4 and 0.8.

values of $\hat{\varepsilon}^{\xi}_{i,t}$. This is based on the assumption that at any given time only a relatively small fraction of Compustat firms is financially constrained. Nonetheless, we analyse the robustness of the results to values of τ that select the 50% and 90% highest values of $\hat{\varepsilon}^{\xi}_{i,t}$. We proceed in a similar way to construct the alternative indicators defined in equations (42) and (43). Finally, notice for some empirical applications it might be more appropriate to select a threshold that is year-specific, so that τ_t selects the most constrained firms in a given year t. This is the case for example of the great recession experiment explained in the next section.

4.2.1 The great recession as a natural experiment

In this section, we verify the validity of our indicators of financial frictions with a quasi natural experiment setting, using the 2008-2009 financial crisis. One notable feature of that crisis, emphasised in a wide empirical literature, is that it was sudden and unexpected. Initial problems in financial markets became apparent during the second half of 2007, but only during 2008 financial and economic conditions began to deteriorate rapidly. The fact that the crisis was so sudden and unexpected has been exploited by many researchers to construct quasi-natural experiments where the treated sample is a group of firms more likely to be affected by the crisis because of exogenous reasons.

An example is Chodorow-Reich (2014), who exploits the fact that the financial crisis affected asymmetrically lenders depending on their exposure to the subprime market. The author argues that firms chose lenders in the pre-crisis period without knowing or evaluating the danger of such exposure. In his case, the treated sample are firms that were borrowing from lenders which during the 2008-2009 crisis were more adversely affected because of such exposure.

We propose here a pseudo-natural experiment based on the following identification strategy. Define $\hat{\varepsilon}^{\xi}_{i,2007}$ as the financial friction shock for firm i in year 2007 (computed using only balance sheet data up to 2007), and $\widehat{\mathcal{I}}^{\xi}_{i,2007}$ as the associated financial frictions indicator. Firms selected with $\widehat{\mathcal{I}}^{\xi}_{i,2007} = 1$ are facing difficulties in accessing external finance *before* the crisis, and hence we expect them to be more negatively affected by the onset of the financial crisis in 2008 than the other firms, because in that period, given the sudden freeze of financial markets, it was practically impossible to find alternative sources of financing.

More specifically, our hypothesis is that firms identified at the end of 2007 as financially constrained will reduce their employment in 2008 relatively more than the other firms, more so than the difference in employment decisions between constrained and unconstrained firms for any year t before 2007.

To implement this test, we run a regression where the dependent variable is the log of employment in period t. Among the regressors, we include the lagged dependent variable, several lagged control variables, and the lagged financial constraints indicator $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$. All regressors are also interacted with the dummy Gr, equal to one for the year 2008 and equal to zero for the previous years. Therefore, the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$ indicates whether the most constrained firms in year t-1 have an employment policy in year t different from the other firms, and the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ indicates whether this relation is different when $t = 2008$ than when $t < 2008$.

Our hypothesis is that the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ is negative and significant. We run these regressions on a balanced sample starting in 1992, and therefore comprising all firms that were continuously in operation in the 16 years before 2008. This excludes the youngest firms and thus likely weakens our test, but makes the estimation of the $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ coefficient more meaningful, because it compares the year 2008 with a relatively long period which includes both past recessionary and expansionary periods.[23] Importantly, for the estimation of $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$ we only consider information up to 2007.

Table 1 shows the estimation results. Column 1 includes as independent variables those described above plus sector-year and firm fixed effect. The coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$ is not significant and very close to zero, indicating that over the whole period there is no systematic relation between the financial friction indicator in period $t-1$ and employment growth in the next period. This result is consistent with the widely held view that, in normal times, large public US companies are subject to few financial frictions. Conversely, $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ is quantitatively large, negative, and significant, indicating that the group of most financially constrained firms in 2007 contracted their employment in 2008 by around 6.7% more than the other firms. This result is robust to sector-year fixed effect. In the other columns of this table we introduce additional control variables that rule out alternative explanations. Since firms with $\widehat{\mathcal{I}}^{\xi}_{i,t-1} = 1$ are in part identified with declines in output and inputs, the indicator might simply capture unproductive firms. Therefore, in Column 2 we include an alternative indicator $\widehat{\mathcal{I}}^{z}_{i,t-1}$ which is equal to one for the 25% of firms with lowest productivity shock $\hat{\varepsilon}^{z}_{i,t-1}$ and zero otherwise. The coefficient of $\widehat{\mathcal{I}}^{z}_{i,t-1}$ is negative, indicating that less productive firms in year $t-1$ tend to reduce employment in the next period. Its interaction with the Gr dummy is also negative, but more importantly the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ remains large, negative, and significant, consistently with our hypothesis that it captures financially constrained rather than unproductive firms. In Column 3, we control for other characteristics that are normally related to financial frictions, such as dummies selecting the 25% smallest, least productive and with highest leverage firms in period $t-1$. In Column 4 we also include labour productivity and leverage in period $t-1$ in levels. All these variables are also interacted with the dummy Gr. We find that controlling for all these characteristics changes

[23]Furthermore, to limit the possibility that results are driven by abnormal changes in employment caused by exceptional events, we exclude the 1% tails of the distribution of yearly employment changes.

little the magnitude of the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$. In other words, our measure of financial frictions does not simply capture factors such as leverage, size and productivity. Finally, in the last column of Table 1 we substitute the indicator $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ with the actual value of $\hat{\varepsilon}^{\xi}_{i,t-1}$. Its interaction with the Gr dummy is also negative and significant, confirming the previous result.

In order to address the concern that the results show in Table 1 might have been obtained by chance, Table 2 performs a placebo experiment. We repeat the estimation shown above in column (2) of Table 1, but in a different time frame. The first column, denoted with "2008", is the experiment described above, in which the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ measures the effects of financial frictions in 2007 for employment growth in 2008. The next column (2) is a "2007" placebo experiment in which we repeat the same procedure described above on a balanced sample of firms with data up to 2007. In this case the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ measures the effects of financial frictions in 2006 for employment growth in 2007. In the following columns (3) to (8) we consider placebo experiments from 2001 to 2006. Since none of these years witnessed a financial shock as large as the one in 2008, we should find the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ to be smaller with respect to the one in column 1, and indeed we find that this is the case. In columns (2)-(8), the coefficient is either not statistically significant, or marginally significant, and always much smaller than in the first column. This evidence reinforces our conclusion that $\widehat{\mathcal{I}}^{\xi}_{t}$ is a valid measure of financial frictions.

Table 3 considers the alternative indicators described in equations (42) and (43). Columns (1) and (3) are analogous to column 1 in Table 1, but using the alternative indicators $\widehat{\mathcal{I}}^{\frac{y}{l}}_{i,t-1}$ and $\widehat{\mathcal{I}}^{b}_{i,t-1}$, respectively, as indicators of financial frictions. Furthermore Columns (2) and (4) consider the continuous values $\hat{b}^{\xi}_{i,t-1}$ and $(\frac{\hat{y}}{l})^{\xi}_{i,t-1}$ and are therefore analogous to column (5) of Table 1. We find that these alternative indicators generate qualitatively similar results than our benchmark indicator $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$.

Finally, in Appendix S5 we provide some additional robustness checks. Table S4 considers alternative values of the threshold τ to compute the financial frictions indicator (see equation 41), and we find that the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ is always negative and significant, and is larger the higher the threshold is. Table S5 considers alternative regressions in which $\widehat{\mathcal{I}}^{\xi}_{i,t}$ is computed using the lower and upper bound values of the identified set of $\hat{\varepsilon}^{\xi}_{i,t-1}$, rather than the median value used for Table 1, and finds similar results, with the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ being negative and statistically significant, even though slightly smaller in magnitude.

4.2.2 Consistency with narrative indicators

In this section, we compare our indicators of financial frictions with those constructed using narrative methods. In particular, we consider the indicator proposed by Hoberg and Maksimovic (2014). These authors perform an automatic text analysis of the "Liquidity and

Table 1: Financial frictions in 2007 and employment contraction in 2008

	(1)	(2)	(3)	(4)	(5)
$\log(l_{t-1})$	0.896***	0.888***	0.888***	0.891***	0.894***
	(99.962)	(97.533)	(100.554)	(102.903)	(101.970)
$\log(l_{t-1}) * Gr$	-0.006	-0.005	0.006	0.005	0.007
	(-1.577)	(-1.451)	(1.230)	(1.131)	(1.533)
$\mathcal{I}^{\xi}_{t-1}$	0.003	0.006	0.002	-0.004	
	(0.575)	(1.087)	(0.451)	(-0.680)	
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.067***	-0.062***	-0.061***	-0.059***	
	(-3.967)	(-3.579)	(-3.432)	(-3.142)	
$\mathcal{I}^{Z}_{t-1}$		-0.035***	-0.033***	-0.030***	
		(-6.510)	(-6.271)	(-5.705)	
$\mathcal{I}^{Z}_{t-1} * Gr$		-0.016	-0.018	-0.016	
		(-0.862)	(-0.961)	(-0.861)	
$Small_{t-1}$			-0.043	-0.039	-0.026
			(-1.260)	(-1.201)	(-0.929)
$Small_{t-1} * Gr$			0.062***	0.060***	0.066***
			(2.713)	(2.621)	(2.798)
$Highlev_{t-1}$			-0.025***	-0.014*	-0.014*
			(-3.443)	(-1.743)	(-1.752)
$Highlev_{t-1} * Gr$			0.003	-0.008	0.002
			(0.200)	(-0.425)	(0.090)
$Lowprod_{t-1}$			-0.021**	-0.014	-0.015
			(-2.256)	(-1.371)	(-1.543)
$Lowprod_{t-1} * Gr$			0.021	0.046**	0.034*
			(1.242)	(2.362)	(1.700)
$labp_{t-1}$				0.022*	0.036***
				(1.730)	(2.766)
$labp_{t-1} * GR$				0.028*	0.025*
				(1.788)	(1.673)
lev_{t-1}				-0.010**	-0.016***
				(-2.357)	(-3.512)
$lev_{t-1} * GR$				0.007	0.005
				(1.147)	(0.677)
$\hat{\varepsilon}^{\xi}_{t-1}$					-0.011***
					(-3.548)
$\hat{\varepsilon}^{\xi}_{t-1} * Gr$					-0.022***
					(-3.002)
Obs.	6,710	6,710	6,710	6,710	6,710
R^2	0.858	0.859	0.860	0.860	0.860
Number of firm	506	506	506	506	506

Notes: The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects and sector-year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}^{\xi}_{t-1}$ is a dummy variable equal to one for the upper quartile of the financial friction shock $\hat{\varepsilon}^{\xi}_{t-1}$ in year $t-1$ and zero otherwise. $\widehat{\mathcal{I}}^{Z}_{t-1}$ is a dummy variable equal to one for the lower quartile of the productivity shock $\hat{z}_{t-1}$ in year $t-1$ and zero otherwise. Gr is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. lev_{t-1} is total debt over fixed assets, and $labp_{t-1}$ is labour productivity (measured as real output divided by number of employees). The variables $highlev_{t-1}$ and $lowprod_{t-1}$, are equal to one for the 25% firm-year observations with highest leverage and lower labour productivity in year $t-1$, respectively, and equal to zero otherwise. $Small_{t-1}$ is a dummy variable if the firm belongs to the quartile of smallest firms in period $t-1$, and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

Table 2: PLACEBO TEST

VARIABLES	(1) 2008	(2) 2007	(3) 2006	(4) 2005	(5) 2004	(6) 2003	(7) 2002	(8) 2001
$\mathcal{I}^{\xi}_{t-1}$	0.006	0.004	0.009**	0.009**	0.008*	0.004	0.003	0.002
	(1.087)	(0.892)	(2.110)	(2.135)	(1.805)	(0.925)	(0.706)	(0.356)
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.062***	-0.010	-0.022	-0.021	-0.010	-0.030*	-0.005	0.005
	(-3.579)	(-0.600)	(-1.277)	(-1.305)	(-0.748)	(-1.827)	(-0.285)	(0.239)
Obs.	6,710	6,880	7,125	7,435	7,632	7,802	7,957	8,090
R^2	0.859	0.858	0.857	0.856	0.860	0.853	0.850	0.847
Number of firm	506	519	537	563	577	591	603	613

Notes: The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects and sector-year fixed effects are included in all specifications. Column (1) replicates the results from Column (1) in Table 1. See the footnote of Table 1 for details. Column (2) is a "2007" placebo experiment in which we repeat the same procedure described above on a balanced sample of firms with data from from 1990 to 2007, instead of from 1991 to 2008. In this case the coefficient of $\widehat{\mathcal{I}}^{\xi}_{t-1} * Gr$ measures the effects of financial frictions in 2006 for employment growth in 2007. Columns (3)-(8) are placebo experiments in which the coefficient of $\widehat{\mathcal{I}}^{\xi}_{t-1} * Gr$ measures the effects of financial frictions in 2000-2005 for employment growth in 2001-2006, respectively. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

Capital Resources" section of the 10-K reports of public US firms. They look for a set of keywords that are related to "delay", and that are relatively close in the text to another set of keywords related to investment plans, and derive a quantitative index, called *delaycon*, that measures the likelihood firms signal the intention to delay their investment plans because of liquidity issues. The index measures the severity of this problem in the cardinal sense, meaning that it can be used to rank firms according to the severity of their constraints (both cross sectionally and over time), and it is publicly available.[24]

Further, the authors refine this indicator with additional information on the firms financing plans. The indicator *debtdelaycon* is high if additional keywords indicate the firms will likely address these liquidity problems by issuing debt. Conversely, the indicator *equitydelaycon* signals the firms will likely address them by issuing equity.

In terms of the power of the *delaycon* indicator to detect financially constrained firms, the authors argue that because the keywords related to delay and investment are mentioned in the "Capitalization and Liquidity" subsection, it follows by context that these firms are delaying investment due to liquidity problems. However lacking an explicit causal link from liquidity issues to capital investment decisions, the concern remains that the liquidity problems might be related to timing issues rather than financial constraints that significantly raise the cost of external finance.

In this respect, we think the subindex *equitydelaycon* is a more precise indicator, because the well-known pecking order theory implies that issuing equity is on average more costly

[24]See http://faculty.marshall.usc.edu/Gerard-Hoberg/MaxDataSite/index.html. The authors initially define a set of firms identified as financially constrained while the other firms are considered not financially constrained. then they construct a continuous version of this indicator using cosine similarity methods.

Table 3: FINANCIAL FRICTIONS IN 2007 AND EMPLOYMENT CONTRACTION IN 2008 - ALTERNATIVE INDICATORS

	(1)	(2)	(3)	(4)
$\log(l_{t-1})$	0.904***	0.906***	0.902***	0.906***
	(98.981)	(90.676)	(98.012)	(90.411)
$\log(l_{t-1}) * Gr$	-0.005	-0.005	-0.005	-0.006
	(-1.347)	(-1.461)	(-1.319)	(-1.476)
$\mathcal{I}^{\frac{y}{l}}_{t-1}$	0.020***			
	(3.549)			
$\mathcal{I}^{\frac{y}{l}}_{t-1} * Gr$	-0.041**			
	(-2.541)			
$(\hat{\frac{y}{l}})^{\xi}_{t-1}$		0.131***		
		(3.265)		
$(\hat{\frac{y}{l}})^{\xi}_{t-1} * Gr$		-0.272***		
		(-4.032)		
$\mathcal{I}^{b}_{t-1}$			0.014**	
			(2.476)	
$\mathcal{I}^{b}_{t-1} * Gr$			-0.038**	
			(-2.317)	
$\hat{b}^{\xi}_{t-1}$				0.014***
				(3.225)
$\hat{b}^{\xi}_{t-1} * Gr$				-0.030***
				(-3.898)
Obs.	6,710	6,710	6,710	6,710
R^2	0.858	0.858	0.858	0.858
Number of firm	506	506	506	506

Notes: The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects and sector-year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}^{\frac{y}{l}}_{t-1}$ is equal to one if the value of $(\hat{\frac{y}{l}})^{\xi}_{t-1}$, which is the part of labour productivity explained by past financial frictions shocks, is larger than the 75th percentile and zero otherwise. $\widehat{\mathcal{I}}^{b}_{t-1}$ is equal to one if $\hat{b}^{\xi}_{t-1}$, which is the part of debt explained by the past financial frictions shocks is smaller than the 25th percentile, and zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

than issuing debt. Therefore a firm identified as having liquidity problems in financing investment, and planning to issue equity, is likely to be constrained in issuing debt, which should be correlated to a positive financial frictions shock $\varepsilon^{\xi}_{i,t}$. Conversely, the subindex *debtdelaycon* is potentially problematic for our purpose, since a firm that is able to issue debt to address liquidity problems is not likely to be substantially financially constrained. The validity of the subindex *equitydelaycon* is also highlighted by the authors who report that it is more strongly correlated to their narrative evidence on investment delay than *debtdelaycon*. Furthermore, *equitydelaycon* is positively correlated to the Whited and Wu (2006) index of financial frictions, more so than the main *delaycon* index, while *debtdelaycon* is negative correlated (see Table 2 in Hoberg and Maksimovic (2014)).

Another issue with this indicator is that by construction it might mostly capture firms constrained in fixed capital investment, because of the type of keywords used by the authors, which focus on expansion plans and capital investment (see Hoberg and Maksimovic 2014, page 1321). Therefore, firms that do not have expansion plans but are constrained and forced to reduce their variable inputs such as wages and materials, might not be captured by this indicator.

This is not to say that the narrative indicator *equitydelaycon* is flawed, rather that by design it likely focuses on constraints to fixed capital expansion, differently from our measure, which can potentially capture financial frictions affecting both expanding and contracting firms. We control for this potential difference by performing regressions in which we only include firm-year observations in which firms did not reduce their stock of fixed capital.

In Table 4 we show regression results where the dependent variable is *equitydelaycon* and the main explanatory variable is our benchmark financial frictions indicator $\widehat{\mathcal{I}}^{\xi}_{i,t}$. Standard errors are clustered at the firm level, and all regressions include Sector-Year fixed effects. In column 1, we consider the full sample, which comprises all manufacturing firms with at least 3 observations in the 1997-2015 period (the years for which the narrative data is available). In column 2 we consider the sub-sample that excludes firm-year observations in which firms reduced their stock of fixed capital. And in column 3 we consider the complementary sub-sample. The coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t}$ is positive and significant for the whole sample, and columns 2 and 3 show that the correlation is entirely driven by periods in which the firms did not reduce their stock of fixed assets, which is consistent with the view that *equitydelaycon* is mainly capturing expansion constraints. Columns 3-6 repeat the same regressions in columns 1-3, adding the same control variables added in Table 1 plus a variable that measures firms size (log of fixed assets). The coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t}$ becomes smaller in magnitude but it is still statistically significant for the non-contracting firms in column 5.

Table 5 repeats the analysis of the first three columns of Table 4 for different thresholds to compute $\widehat{\mathcal{I}}^{\xi}_{i,t}$. Its coefficient more than doubles when moving from the benchmark indicator

Table 4: Consistency with narrative indicators of financial frictions.

	(1)	(2)	(3)	(4)	(5)	(6)
$\widehat{\mathcal{I}}_t^{\xi}$	0.003*	0.007***	0.000	0.002	0.006**	-0.001
	(1.663)	(2.630)	(0.077)	(1.023)	(2.260)	(-0.473)
$small_t$				0.028***	0.026***	0.031***
				(5.202)	(4.650)	(4.486)
$highlev_t$				0.000	-0.000	0.006
				(0.107)	(-0.100)	(1.201)
$lowprod_t$				0.018***	0.017***	0.021***
				(4.958)	(4.308)	(4.201)
$log(fixedassets_t)$				0.002	0.001	0.002
				(1.224)	(0.778)	(1.137)
lev_t				0.001	-0.000	0.001
				(1.120)	(-0.265)	(1.278)
$labp_t$				-0.000	-0.000	-0.000
				(-0.616)	(-0.356)	(-0.715)
Obs.	13,697	8,147	5,550	13,557	8,063	5,494
R^2	0.072	0.097	0.078	0.096	0.117	0.112

Notes: The table shows the results from regressing a narrative indicator on financial frictions on the indicator computed using our procedure. The dependent variable *equitydelaycon* is a quantitative indicator which is higher the more likely the firm is to delay investment decisions because of liquidity problems, which the firm plan to address them by issuing equity (see Hoberg and Maksimovic (2014) for details). Among the regressors, $\widehat{\mathcal{I}}_t^{\xi}$ is a dummy variable equal to one for the upper quartile of the financial friction shock $\hat{\varepsilon}_t^{\xi}$ in year t and zero otherwise. lev_t is total debt over fixed assets, and $labp_t$ is labour productivity (measured as real output divided by number of employees). The variables $highlev_t$ and $lowprod_t$, are equal to one for the 25% firm-year observations with highest leverage and lower labour productivity in year $t-1$, respectively, and equal to zero otherwise. $Small_t$ is a dummy variable if the firm belongs to the quartile of smallest firms in period t, and equal to zero otherwise. All specifications include a constant and sector-year fixed effects. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

(middle panel) to the one that identifies financially constrained firms with those with a financial frictions shock above the 90th percentile (lower panel). One plausible explanation is that in normal times few Compustat firms are financially constrained, and the 90th percentile is a more precise threshold to identify them. In terms of magnitude, the interquartile range of the dependent variable *equitydelaycon* is equal to 0.1. Therefore, with respect to this range, belonging to the most constrained firms according to column 2 (lower panel), increases the value of the narrative indicator by 16%.

4.3 Quasi-universe of Spanish manufacturing firms

The evidence in the previous sections, based on Compustat data, has shown that our financial frictions indicator is consistent with alternative ways to identify financially constrained firms.

Table 5: Consistency with narrative indicators of financial frictions, alternative thresholds.

$\widehat{\mathcal{I}}_t^{\xi} = 1$ if $\varepsilon_t^{\xi} > 50\%$

	(1)	(2)	(3)
$\widehat{\mathcal{I}}_t^{\xi}$	0.001	0.003	-0.001
	(0.815)	(1.501)	(-0.424)
Obs.	13,697	8,147	5,550
R^2	0.072	0.097	0.078

$\widehat{\mathcal{I}}_t^{\xi} = 1$ if $\varepsilon_t^{\xi} > 75\%$(Benchmark)

	(1)	(2)	(3)
$\widehat{\mathcal{I}}_t^{\xi}$	0.003*	0.007***	0.000
	(1.663)	(2.630)	(0.077)
Obs.	13,697	8,147	5,550
R^2	0.072	0.097	0.078

$\widehat{\mathcal{I}}_t^{\xi} = 1$ if $\varepsilon_t^{\xi} > 90\%$

	(1)	(2)	(3)
$\widehat{\mathcal{I}}_t^{\xi}$	0.012***	0.016***	0.009**
	(3.829)	(3.506)	(2.173)
Obs.	13,697	8,147	5,550
R^2	0.073	0.099	0.080

Notes: The table shows the results from regressing a narrative indicator on financial frictions on the indicator computed using our procedure. The dependent variable *equitydelaycon* is a quantitative indicator which is higher the more likely the firm is to delay investment decisions because of liquidity problems, which the firm plan to address them by issuing equity (see Hoberg and Maksimovic (2014) for details). $\widehat{\mathcal{I}}_t^{\xi}$ is a dummy variable equal to one for the upper quartile of the financial friction shock $\hat{\varepsilon}_t^{\xi}$ in year $t-1$ and zero otherwise. All specifications include a constant and sector-year fixed effects. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

Importantly, it has also shown that it is able to identify both constraints that cause firms to contract their activity, as in the great recession experiment, and those that cause firms to limit their expansion, as those identified by the narrative indicator of Hoberg and Maksimovic (2014).

While useful for comparison purposes, it must be noted that Compustat is a sample of large public companies, and therefore it is interesting to verify our methodology also on more comprehensive data that include smaller firms. Indeed one of the main motivations of this paper is to provide a new indicator that can be computed for the latter group, since it requires only balance sheet data. Therefore, in this section we apply our methodology on a sample of Spanish manufacturing firms from SABI (Sistema de Análisis de Balances Ibéricos), which includes the quasi-universe of Spanish firms with more than 5 employees. After applying some basic filtering (dropping firms-year observations with missing assets and revenues and employment data), we obtain a sample of around 45.000 observations every year for the 2000-2018 period, for a total of more than 860.000 observations. 67% of all the observations are on firms smaller than 20 employees, 31% on firms between 20 and 250 employees, and 1.6% on firms larger than 250 employees.

We first perform the Great Recession Exercise in Table 1 also on the Spanish sample, obtaining similar results to those of the Compustat sample. Detailes are in the Appendix, see Table S6 in Section S5. Here we are interested in describing two additional tests that take advantage of the larger heterogeneity of the Spanish sample. In paritcular, we focus on the magnitude and persistence of the effects of financial frictions shocks, verifying the following hypotheses:

Hypothesis 1: If $\hat{\varepsilon}^{\xi}_{i,t}$ measures financial frictions shocks, we expect it to explain a larger part of employment volatility for small relative to large firms.

Hypothesis 2: If $\hat{\varepsilon}^{\xi}_{i,t}$ measures financial frictions shocks, we expect its effect to be more persistent for small relative to large firms.

The intuition for the first hypothesis is that smaller firms are more likely to be subject to credit shocks, because of a “flight to quality” effect. When a lender is constrained in its funding, it will prioritise lending to its larger customers, insulating them from fluctuations in the availability of credit, and leaving the smaller customers more exposed.

The intuition for the second hypothesis is that large firms not only have better access to their main lender, but also relations with multiple banks. If in a given period the firm is forced to reduce its employment level because of unexpected problems in accessing external financing, it is expected that it will be able to find alternative sources relatively quickly. Conversely, many smaller firms typically rely only on one main lender. If they face an increase in financing problems, they will be much less likely to find quickly suitable alternatives.

To test Hypotheses 1 and 2, we measure to what extent financial frictions shocks $\hat{\varepsilon}^{\xi}_{i,t}$ explain employment variations at different time horizons. In practice we run several panel regressions with firm fixed effects in which $\hat{\varepsilon}^{\xi}_{i,t}$ is the main explanatory variable, and the dependent variable is the log of employment at different time horizons. The statistic we use to verify the hypotheses is the *within* R^2 statistic, which measures the share of within-firm variability in the dependent variable explained by $\hat{\varepsilon}^{\xi}_{i,t}$. For this purpose, both dependent and independent variables are are deviations from 2digit sector-year averages, to ensure the within R2 statistic is not affected by sector-year factors.

The regressions results are shown in Panel A of Table 6. For completeness, in addition to the *within* R^2 statistic we also report the standardized regression coefficient of $\hat{\varepsilon}^{\xi}_{i,t}$.[25] For

[25]We standardize the coefficients by subtracting firm-level means from both dependent and independent variables and then dividing by the group level standard deviations, where the groups are the three size categories of firms. This standardization procedure is irrelevant for the within R^2 statistic. In a robustness exercise, not reported here, we also subtract from both dependent and independent variables the sector-year

comparison, Panel B of Table 6 shows the same statistics for the productivity shock $\hat{\varepsilon}^z_{i,t}$. In both cases we report regression results for all firms and for different size groups.

The results relative to all firms show that on impact (first column) the financial frictions shock $\hat{\varepsilon}^\xi_{i,t}$ explains a larger fraction of the variation in employment than the productivity shock $\hat{\varepsilon}^z_{i,t}$ (14.2% versus 6.5%). This difference is entirely driven by the smaller group of firms (14.2% versus 6.4%), while the opposite is true for larger firms. Looking at the effect on impact for firms larger than 500 employees, the productivity shocks explains a larger fraction of the variation in employment than the financial frictions shocks (21% versus 10%), confirming Hypothesis 1. Notice that this does not imply that productivity or demand improvements are not important for the employment growth of small firms. In our procedure for estimating the shocks we control for year fixed effects, thus abstracting from common long run growth factors.

Looking at the other columns, we see that on average for the whole sample the productivity shocks tend to be more persistent and to explain a larger fraction of the variation in employment at longer horizons than the financial shock. Intuitively, productivity shocks have, by their nature, persistent effects. Think for example about a firm that introduces a new line of products. This boosts revenues in the short term and the effect is expected to be persistent, as the firm builds a customer base for the new products. Conversely, financial shocks are less persistent because generate costly distortions, and the firms act (by finding alternative financing sources, or increasing retained earnings) to mitigate their long run effects. Importantly, Panel A shows that the effects of financial friction shocks are more persistent for smaller than for larger firms, consistently with Hypothesis 2.

5 Conclusion

We develop a novel approach to identify and estimate financial shocks and the intensity of financial frictions at the firm level. Our methodology is inspired by the sign-based identification literature for aggregate time series (e.g. Uhlig 2005). Its key advantage is that it is easy to implement, and that it relies on a limited set of identifying restrictions. In particular we show that, imposing only mild assumptions, which are consistent with a wide class of theoretical models, we can simultaneously recover firm's productivity, profitability and financial shocks and the latent intensity of financial frictions. We validate our methodology on simulated data from the structural model. Furthermore, we apply it on both Compustat data and on the quasi universe of Spanish manufacturing firms above 5 employees, providing several empirical tests of its validity.

effects, and then we repeat the analysis, finding very similar results to those reported in Table 6.

Table 6: Local Projections of the effect of shocks on small and large firms.

	(1)	(2)	(3)	(4)	(5)	(6)
	$\log(l_t)$	$\log(l_{t+1})$	$\log(l_{t+2})$	$\log(l_{t+3})$	$\log(l_{t+4})$	$\log(l_{t+5})$
Panel A, independent variable is $\hat{\varepsilon}^{\xi}_{i,t}$						
All firms	-0.316***	-0.269***	-0.199***	-0.143***	-0.095***	-0.055***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
within R^2	0.142	0.106	0.0595	0.0311	0.0143	0.00496
Less than 250 employees	-0.317***	-0.270***	-0.200***	-0.143***	-0.095***	-0.055***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
within R^2	0.142	0.106	0.0597	0.0311	0.0144	0.00499
between 250 and 500 empl.	-0.246***	-0.219***	-0.151***	-0.128***	-0.076***	-0.034**
	(0.028)	(0.024)	(0.021)	(0.022)	(0.016)	(0.013)
within R^2	0.112	0.0894	0.0473	0.0336	0.0125	0.00253
Larger than 500 empl.	-0.239***	-0.172***	-0.106***	-0.037	-0.012	0.030
	(0.042)	(0.041)	(0.033)	(0.025)	(0.026)	(0.023)
within R^2	0.100	0.0510	0.0212	0.00277	0.000299	0.00202
Panel B, independent variable is $\hat{\varepsilon}^{z}_{i,t}$						
All firms	0.217***	0.221***	0.191***	0.158***	0.125***	0.091***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
within R^2	0.0648	0.0676	0.0511	0.0352	0.0227	0.0124
Less than 250 employees	0.217***	0.221***	0.191***	0.158***	0.125***	0.091***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
within	0.0644	0.0673	0.0509	0.0351	0.0227	0.0124
between 250 and 500 empl.	0.251***	0.229***	0.187***	0.149***	0.101***	0.069***
	(0.030)	(0.028)	(0.023)	(0.022)	(0.016)	(0.015)
withinR^2	0.112	0.0953	0.0701	0.0435	0.0218	0.0100
Larger than 500 empl.	0.345***	0.328***	0.249***	0.177***	0.090***	0.044*
	(0.043)	(0.047)	(0.044)	(0.032)	(0.027)	(0.026)
withinR^2	0.210	0.185	0.111	0.061	0.016	0.004

Notes: All variables are deviations from 2digit sector-year averages. The dependent variable is the logarithm of the number of employees. From the first to last column, the time horizon increases from 0 years, with $\log(l_t)$ as dependent variable, to 5 years, with $\log(l_{t+5})$ as dependent variable. Firm fixed effects are also included. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

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