Policy evaluation with Sufficient Macro Statistics*

— A PRIMER —

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Abstract

Impulse responses and forecasts are central concepts for policy makers. In addition, they are also sufficient statistics to solve many important macroeconomic problems, from policy counterfactuals to policy evaluation, and they offer a promising alternative to the standard structural modeling approach. In this review paper, we discuss and extend recent progress on the use of these sufficient macro statistics for policy evaluation. We illustrate the methods by evaluating the performance of the ECB over 1999-2023.

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1 Introduction

How should we evaluate the latest policy decision by the Fed or the ECB? How should we compare the performances of the Fed vs the ECB at handling the 2007-2008 crisis? And more generally, how should we evaluate the performance of an elected official in office?

Answering these questions is at the core of good macroeconomic policy making, but until recently there had been surprisingly little work on macroeconomic policy evaluation per se. Part of this state of affairs owes to the possibility of model mis-specification. Despite impressive recent progress in structural macro modeling, the underlying economy is so complex that quantitative conclusion drawn from a specific model may be too uncertain to reach any form of consensus.

In recent years, several papers have demonstrated how sufficient statistics can be applied in macroeconomics; for policy evaluation and policy guidance, for policy counterfactuals, or for the elicitation of policy makers' preferences.[1] This "sufficient macro statistics" approach requires minimal assumptions on the underlying structural economic model, and instead relies on recent advances of econometrics; identification and forecasting (e.g., Elliot and Timmermann, 2016; Ramey, 2016)

In this review paper, we summarize some of the main lessons of that literature in the context of policy evaluation, and we illustrate the approach with an evaluation of ECB policy since its inception in 1999.

An illustration

To illustrate the key features of macro policy evaluation with sufficient macro statistics, we start with a concrete example.

The policy problem

Consider a policy maker, think for instance of a central banker, who assumes office at $t = 0$ and at each period $t \geq 0$ must decide on an expected policy path $\mathbb{E}_t \mathbf{P}_t$ to minimize the loss function

$$\mathcal{L}_t = \mathbb{E}_t \left(\mathbf{Y}_t - \mathbf{Y}^*\right)' \mathcal{W} \left(\mathbf{Y}_t - \mathbf{Y}^*\right), \tag{1}$$

where $\mathbb{E}_t \mathbf{P}_t$ is the time-t expected path of the policy instrument $(p_t, p_{t+1}, ...)$, $\mathbb{E}_t \mathbf{Y}_t$ is the expected path of macro variables of interest, for instance inflation and unemployment, and the vector $\mathbf{Y}^*$ is the policy maker's target values for the path $\mathbf{Y}_t$, for instance an inflation target of 2 percent and an unemployment target of 4 percent. $\mathcal{W}$ is a weighting matrix.

[1] See Barnichon and Mesters (2022, 2023*b*,*a*); Beraja (2023); McKay and Wolf (2022, 2023); Caravello, McKay and Wolf (2024); de Groot et al. (2021); Hebden and Winkler (2021).

When setting her policy path $\mathbb{E}_t\mathbf{P}_t$, the policy maker follows a generic Taylor-type rule[2]

$$\mathbb{E}_t\mathbf{P}_t = \Phi(\mathbb{E}_t\mathbf{Y}_t - Y^*) + \boldsymbol{\varepsilon}_t \ , \tag{2}$$

where Φ captures the systematic response of $\mathbb{E}_t\mathbf{P}_t$ to the economic outlook, and $\boldsymbol{\varepsilon}_t$ is a stochastic component capturing mistakes, exogenous deviations from the rule. Expression (2) highlights how a policy maker's performance will depend on two elements that we collect in a policy rule vector $\phi = \{\Phi, \sigma_\epsilon\}$:[3]

1. The systematic reaction to business cycle fluctuations and shocks affecting the economy (Φ): For example, a bad policy maker could react too strongly/weakly to inflation.

2. Erratic deviations from the policy rule (the variance of policy shocks σ_ε): A bad policy maker could make frequent/large random mistakes.

Measuring performance

Macroeconomic policy making differs from other policy making problems in that macroeconomic policy decisions are typically (i) sequential and (ii) dynamic. For instance, central bankers convene at regular intervals during the year to decide on the policy path going forward, and fiscal policy makers decide each year on a budget path going forward. In other words, while in office a typical macroeconomic policy maker will have to make a *sequence* of choices about her expected policy *path*. In that context, two policy problems can be considered and evaluated: (i) a time-t problem whereby a policy maker must decide on a policy path given the state of the economy *today* —this is about appropriately setting policy on one of these decision dates—, (ii) a timeless problem whereby the policy maker must decide on a reaction function to minimize her expected loss over the sequence of choices she will have to make. While these two problems are closely related, they carry different implications, and we will see how the sufficient macro statistics approach can be used to evaluate both problems.

[2]Rule (2) reduces to a standard Taylor rule in the baseline New-Keynesian model (e.g. Galí, 2015).

[3]There is a third policy element that we abstract from; the long-run value of the policy instrument. For instance, a bad policy maker could keep the policy rate systematically too low/high, resulting in inflation running permanently above its 2 percent target. In that case, the long-run value of the policy instrument is inappropriate, i.e., not consistent with the target $\mathbf{Y}^*$. Given our focus on developed economies, we do not incorporate this element of policy performance in our analysis. However, systematically too low or too high policy instruments (or even drifting policy objectives) is an important source of policy failures in developing countries, for instance cases of hyper-inflation or unsustainable debt. Extending our framework to incorporate long-run deviations from targets is an important avenue for future research.

Time-t policy evaluation

Consider first our policy maker as of time t. She has to decide on her expected policy path given the state of the economy. She wishes to follow the prescriptions of the optimal rule, but absent a specific macro model and explicit formulation of the optimal rule, how can she be sure to have identified that optimal path, the path that best balances the different policy objectives? The sufficient macro statistics approach can help, building on the Optimal Policy Perturbation (OPP) statistic introduced in Barnichon and Mesters (2023*b*). Formally, the OPP is

$$\delta_t = -\left(\mathcal{R}'\mathcal{W}\mathcal{R}\right)^{-1}\mathcal{R}'\mathcal{W}\mathbb{E}_t\mathbf{Y}_t \ , \tag{3}$$

where $\mathcal{R}$ captures the impulse responses of the policy objectives to policy shocks under some baseline policy rule ϕ and where $\mathbb{E}_t\mathbf{Y}_t$ captures the forecasts for the policy objectives conditional under that same rule ϕ.

The OPP is the optimal adjustment to the policy rule (2): it is the additive adjustment to the rule that makes the policy path $\mathbb{E}_t\mathbf{P}_t$ optimal, i.e., that minimizes the loss function (1). Essentially, the OPP captures the gradient of the loss function with respect to an innovation to the policy rule. If the gradient is non-zero, it means the rule is not optimal and adjusting the policy path by the rescaled gradient —effectively, a Gauss-Newton method— will deliver the optimal optimal policy path in a linear-quadratic problem like ours.[4]

Note how the OPP involves only the two sufficient macro statistics $\mathbb{E}_t\mathbf{Y}_t$ and $\mathcal{R}$. The first sufficient statistic —the forecasts for the policy objectives— serves to capture the state of the economy at time t —the characteristics of the time-t decision problem— and to define a scenario under a baseline policy rule. The second sufficient statistic —the impulse responses to policy shocks— serves to explore whether deviating from that rule can produce a lower loss. At an optimal policy, the gradient —a weighted product of the two statistics, see (3)— should be zero, and forecasts and impulse responses should be orthogonal: it should not be possible to use the impulse responses to adjust the forecasts and lower the loss function. This orthogonality condition forms the basis of the sufficient statistics approach to policy evaluation.

Importantly, the two sufficient statistics can be estimated using reduced form econometric models. To estimate the impulse responses to policy shocks —the second sufficient statistic— , we can draw a large macroeconometric literature; using reduced form models combined with identification restrictions or instrumental variables (e.g., Ramey, 2016). Alternatively, when econometric evidence is absent one could obtain impulse responses from structural models (e.g. de Groot et al., 2021; Hebden and Winkler, 2021).

[4]Barnichon and Mesters (2023*b*) further show how the OPP statistic can be extended to incorporate non-linearities, policy constraints, e.g. the zero lower bound, and corrections for dynamic inconsistencies (e.g. Barro and Gordon, 1983).

To construct the conditional expectation $\mathbb{E}_t \mathbf{Y}_t$, also referred to as an oracle forecast, we can draw on large forecasting literature (e.g., Elliot and Timmermann, 2016). And while an oracle forecast is an ideal concept, we can approximate it as best as possible: compute the best prediction for the policy objectives under the baseline rule, and take into account the uncertainty associated with that approximation. This is the route proposed in Barnichon and Mesters (2023*b*).[5]

Term policy evaluation

Consider now our policy maker at the beginning of her term. The policy maker's problem is to choose a policy rule vector $\phi = \{\Phi, \sigma_\epsilon\}$ that yields the minimum loss for $\mathbb{E}\mathcal{L}_t$ for any t, so that we can measure a policy maker's performance from the distance to minimum loss Δ given by

$$\Delta = \mathbb{E}\mathcal{L}_t - \mathbb{E}\mathcal{L}_t^{\text{opt}} ,$$

where $\mathbb{E}\mathcal{L}_t$ is the loss under the rule ϕ and $\mathbb{E}\mathcal{L}_t^{\text{opt}}$ is the minium expected loss. Unlike the time-t problem, this is now a timeless perspective meant to capture the problem of a policy maker serving an infinite term.[6]

Absent a specific macro model, how can we measure Δ? Again sufficient statistics can help, though this time we require a sequence of sufficient statistics, with each set corresponding to a specific decision date during the policy maker's term. Specifically, we can compute the distance to minimum loss Δ from

$$\Delta = \mathbb{E}\left(\delta_t^{\Delta\prime} \mathcal{R}' \mathcal{W} \mathcal{R} \delta_t^{\Delta}\right) \qquad \text{with} \qquad \delta_t^{\Delta} = \left(\mathcal{R}' \mathcal{W} \mathcal{R}\right)^{-1} \mathcal{R}' \mathcal{W} \Delta \mathbb{E}_t \mathbf{Y}_t \tag{4}$$

where $\Delta \mathbb{E}_t \mathbf{Y}_t = \mathbb{E}_t \mathbf{Y}_t - \mathbb{E}_{t-1} \mathbf{Y}_t$ is the information update, or forecast update between time $t-1$ and t, such that δ_t^{Δ} is the *OPP innovation*, the innovation to the OPP driven by the new shocks *revealed* at a new decision date. Intuitively, the evaluation of a policy maker's term is based on a simple result: a policy maker follows an optimal rule if and only if she responds optimally to each new set of shocks that she faces during her term.[7]

While expression (4) does not allow to discriminate between the different determinants of

[5] An alternative consists in imposing a high-level assumption about the underlying economic model —a VAR invertibility assumption— in order to compute the oracle forecast, this the route recently proposed in (Caravello, McKay and Wolf, 2024). Yet another alternative to take the policy maker's own published forecast as "the" oracle forecast. Effectively, this approach amounts to evaluating policy makers based on their own (and perhaps mistaken) view of the state of the economy.

[6] The timeless perspective is commonly adopted in structural macro policy literature when aiming to find optimal policy rules (e.g. Woodford, 2003).

[7] And since δ_t^{Δ} captures the deviation from the optimal policy rule coming from new sets of shocks, $\mathcal{R} \cdot \delta_t$ captures the counter-factual correction to the economy's path under the optimal policy. The square of that term then translates that alternative path into units of extra welfare.

policy performance (random mistakes or inappropriate reactions to the state of the economy), Barnichon and Mesters (2023a) show how a third set of sufficient statistics —the impulse responses to non-policy shocks— can allow for such a decomposition. Specifically, we can decompose the distance to minimum loss with

$$\Delta = \underbrace{\sigma_\epsilon^2 \cdot \mathcal{R}'\mathcal{W}\mathcal{R}}_{\text{exog. mistakes}} + \underbrace{\sum_{j=1}^{J} \sigma_{\xi_j}^2 \cdot \tau_j' \mathcal{R}'\mathcal{W}\mathcal{R}\tau_j}_{\text{reaction function}} \quad \text{with} \quad \tau_j = (\mathcal{R}'\mathcal{W}\mathcal{R})^{-1}\mathcal{R}'\mathcal{W}\Gamma_j \ , \tag{5}$$

where J is the number of non-policy shocks, and Γ_j captures the effects of some non-policy shock ξ_{jt} (e.g., an oil shock, a technology shock, a financial shock) on the policy objectives, e.g., inflation and unemployment.

Expression (5) allows to isolate the different determinants of policy performance: (i) inappropriate reaction to shocks (Φ), or (ii) random mistakes (σ_ϵ). While (ii) can be readily measured from the variance of the policy shocks used to estimate $\mathcal{R}$, assessing (i) —the appropriate reaction to non-policy shocks— requires the identification of third set of sufficient statistics Γ_j —the impulse responses to non-policy shocks—, in order to compute the statistic τ_j, the Optimal Reaction Adjustment (ORA) derived in Barnichon and Mesters (2023a).

The ORA τ_j captures by how much more or less the policy maker should have responded to a specific type of non-policy shock. It has a very similar formula as the OPP, where the forecast revision $\Delta\mathbb{E}_t\mathbf{Y}_t$ is replaced by the impulse response to a specific shock, for instance the response to an oil shock. Intuitively, while $\Delta\mathbb{E}_t\mathbf{Y}_t$ captures the total forecast revision coming from the net effect of all the time-t shocks affecting the economy, Γ_j captures the revision to the forecast coming from only one of these time-t shocks. This is nothing but an impulse response. An optimal policy maker should optimally response to any type of forecast revision, and thus to the impulse response of any shocks. This is the condition captured by the ORA. Complementing the OPP, the ORA allows to provide an economic interpretation behind sub-optimal policy rules; by allowing to isolate the type of shock that a policy maker failed to respond to appropriately, for instance an inappropriate systematic response to oil shocks.

Literature review

We briefly place the sufficient macro statistics approach in the broader context of the macroeconomics literature.

Jan Tinbergen was the first to explicitly explore the possibility of evaluating macro policy using a statistical model. In 1936 he completed his work on what became the first empirical macroeconomic model which was designed for the Dutch economy and its purpose was to help the Dutch Central Planning Bureau to develop appropriate economic policies, see Dhaene and Barten (1989). Tinbergen (1952) provides an accessible overview of many of these early ideas. The remarks of Theil (1956) are of particular interest as they highlight concerns about uncertainty and the limited control of policy makers which persist today (e.g. Bénassy-Quéré et al., 2018) and form an important element of the sufficient statistics approach. In the first decades after the war the development of macro econometric models flourished. For instance, in the US, Marschak organized a special team at the Cowles Commission for conducting such analysis, see Bodkin, Klein and Marwah (1991) for an extensive discussion.

Lucas (1976) voiced an important criticism of these models: they ignored that agents in the economy typically adjust their behavior when policy decisions are made, as such models for policy evaluation should allow the state of the economy to depend on the actions of the policy maker, not only through the policy instruments but also via way they shape expectations of agents and perhaps even more structural relationships in the economy. These concerns led a large literature on structural macro economic modeling of which the New Keynesian theories expounded in Woodford (2003) and Galí (2015), as well as their modern heterogeneous agent counterparts (e.g. Auclert, Rognlie and Straub, 2024), are prime examples.

Taking a more econometric perspective Sims (1980) and Sims (1982) explain how structural VAR models can be used for policy analysis. These models are generally smaller in scale when compared to the earlier macro econometric models and rely on more credible identifying restrictions that facilitate the construction of policy counter factuals. Important works that develop this methodology include Sims and Zha (2006), Bernanke et al. (1997), Leeper and Zha (2003) and Antoln-Daz, Petrella and Rubio-Ramrez (2021). While these methods are not fully robust to the Lucas critique they are an essential tool for most policy makers, and the sufficient statistics approach relies on many structural VAR developments.

Broadly speaking, the sufficient statistics approach aims to provide an alternative route for policy analysis that lies between the usage of a fully fledged structural model and the more reduced form structural VAR. An early contribution that explores such *semi-structural* route is Beraja (2023) who notes that several linearized models are observationally equivalent under a benchmark policy rule and yield an identical counterfactual equilibrium under an alternative one. Exploiting this counterfactual equivalence allows to reduce the number of restrictions needed in the structural form while retaining robustness to the Lucas critique.

A further reduction in the number of structural restrictions needed can be obtained for a class of structural models where expected policy paths capture all effects of policy. For this class of models McKay and Wolf (2023) show that the impulse responses to policy news shocks are sufficient statistics for constructing unconditional policy rule counterfactuals that are robust to the Lucas critique. Intuitively, when all effects of policy are transmitted via the expected policy paths the causal effects of such paths policy *at all horizons* allow to replicate the counterfactual effects of any policy rule that induces a unique equilibrium.

Barnichon and Mesters (2023*b*) consider the time-t policy evaluation problem and show that for the same class of models optimal policy paths can be characterized by two sufficient statistics: (i) impulse responses to policy news shocks and (ii) forecasts for the macro variables. In this work we show that their results immediately suggest how to construct arbitrary time-t policy path counterfactuals using the same sufficient statistics. de Groot et al. (2021) and Hebden and Winkler (2021) also consider the time-t problem, but use impulse responses from structural models, and focus more on the algorithms needed for computing policy counterfactuals under various constraints.

To compute policy counterfactuals or optimal policies the previous papers require the identification of policy news shocks at *all* horizons. With only a subset of shocks the results are necessarily approximations of the desired counterfactuals. To improve the approximation Caravello, McKay and Wolf (2024) propose to supplement the evidence from the subset of identified policy shocks with information from structural macro models. Other ways of extrapolating can rely on smoothness restrictions, e.g. B-splines as in de Boor (2001), or factor structures as in Inoue and Rossi (2021). An alternative solution for the missing policy shocks is found in Hack, Istrefi and Meier (2023) who propose a method to directly compute the causal effects of changes in the rule coefficients as opposed to using the causal effects to policy news shocks to replicate changes in the rule coefficients.

While policy counterfactuals and optimal policies are obviously of leading importance in the macro economic toolkit, the sufficient statistics approach can be used for answering other macro questions as well. For instance, Barnichon and Mesters (2023*a*) introduce the ORA statistics for comparing policy makers and institutions after their term. Further, Barnichon and Mesters (2022) show how the framework sketched above can be used to learn policy makers preferences through a revealed preference approach.

More broadly the sufficient macro statistics approach draws inspiration from the sufficient statistics approach in public finance (e.g. Chetty, 2009; Kleven, 2020). Both methods exploit the fact that the welfare consequences of a policy can be derived from high-level elasticities, allowing for policy evaluation without making parametric assumptions or estimating the structural primitives of fully specified models. One feature specific to our macro focus is that the loss function is typically a high level assumption, consistent with the fact that the loss function is often determined by political factors or by statutory requirement. For instance, it is the US Congress that mandates the Federal Reserve to seek stable inflation and full employment. That said, the sufficient macro statistics approach can equally be applied to problems with micro-founded loss functions.

Last, the treatment of uncertainty around the sufficient statistics shares similarities with the robust-control approach to policy making that is outlined in Hansen and Sargent (2008). In particular, parameter and model mis-specification uncertainty are often taken into account when constructing confidence bands around policy recommendations. That said, the decision rules explored by the sufficient macro statistics approach have so far typically focused on minimizing expected loss and have not considered characterizing minimax optimal policies. This is an important avenue for future research.

Paper outline

The remainder of this paper is organized as follows. In the next section we discuss the class of underlying structural models considered. Some useful representations for the equilibrium of the model in terms of the sufficient statistics are shown in Section 3 and the estimation of these statistics is discussed in 4. The methods for time-t and term policy evaluation are discussed Sections 5 and 6, respectively. Section 7 illustrates the methods by evaluating the performances of the ECB over 1999-2023. Section 8 concludes.

2 Structural model

Inspired by Auclert et al. (2021), we adopt a sequence space representation, which is somewhat different from the usual recursive way of writing down dynamic models (e.g. Ljungqvist and Sargent, 2004).[8]

The goal of the sufficient macro statistics approach is to impose minimal assumption on the underlying economic model. Specifically, the only structure that we impose is that the data generating process belongs to a class of generic macro models; that is that the true underlying DGP is a special case of generic model. Importantly, we will not assume that we can learn which specific model generated the data.[9]

Let $\mathbf{Y}_t = (y_t', y_{t+1}', \ldots)'$ denote the time-t path of the macro variables that populate the economy. Specifically, y_{t+h} is an $M_y \times 1$ vector containing the variables of interest at time $t+h$. The policy path is defined by $\mathbf{P}_t = (p_t', p_{t+1}', \ldots)'$, where p_{t+h} is the $M_p \times 1$ vector of policy instruments available at time $t+h$. For instance, a monetary policy maker decides on the path of the overnight interest rate (and possibly on the path additional non-

[8]The Appendix describes in more details the notations and benefits underlying the sequence space representation.

[9]Relaxing this common assumption is at the core of the sufficient macro statistics approach. It allows to considerably sidestep/alleviate the many challenges created by the possibility of model mis-specification, as well as the identification challenges that have plagued the estimation of modern structural macro models (e.g. Canova and Sala, 2009; Andrews and Mikusheva, 2015). We will only need to estimate impulse responses implied by the general model and construct forecasts, both of which need not suffer from the general identification problems that arise in structural macro equations/models. Or at least, there exists a plethora of alternative identification approaches that can be adopted.

standard monetary policy actions, such as bond market purchases), while a government decides on paths for spending, taxes and transfers over the coming years (e.g. Alesina, Favero and Giavazzi, 2019). We assume that all variables have been suitably detrended to be stationary.[10]

Our generic linear model for the economy at time t is given by

$$\mathcal{A}_{yy}\mathbb{E}_t\mathbf{Y}_t - \mathcal{A}_{yp}\mathbb{E}_t\mathbf{P}_t = \mathbf{X}_{-t} + \mathcal{B}_{y\xi}\Xi_t \tag{6}$$

$$\mathcal{A}_{pp}\mathbb{E}_t\mathbf{P}_t - \mathcal{A}_{py}\mathbb{E}_t\mathbf{Y}_t = \mathbf{X}_{-t} + \mathcal{B}_{p\xi}\Xi_t + \mathcal{B}_{p\varepsilon}\boldsymbol{\varepsilon}_t\ , \tag{7}$$

where the pre-determined inputs on the right hand side are the time-t paths of news shocks: $\Xi_t = (\xi'_{t,t}, \xi'_{t,t+1}, \xi'_{t,t+2}, \ldots)'$ and $\boldsymbol{\varepsilon} = (\varepsilon'_{t,t}, \varepsilon'_{t,t+1}, \varepsilon'_{t,t+2}, \ldots)'$, as well as the path of any time-t initial conditions $\mathbf{X}_{-t}$, which includes the effects of all past shocks.

The vector $\xi_{t-j,t+h}$ includes structural shocks that capture the exogenous information about time period $t+h$ but are released at time $t-j$. Similarly, $\varepsilon_{t-j,t+h}$ is the vector of policy news shocks for period $t-h$ that are released at $t-j$. We assume that all news shocks are mean zero with unit variance and mutually and serially uncorrelated.

The linear maps $\mathcal{A}_{..}$ and $\mathcal{B}_{..}$ are infinite dimensional and conformable such that the multiplications are well defined. The conditional expectation operator is defined as $\mathbb{E}_t(\cdot) = \mathbb{E}(\cdot|\mathcal{F}_t)$, where the time-$t$ information set $\mathcal{F}_t$ is defined in terms of the pre-determined inputs, i.e. $\mathcal{F}_t = \{\mathbf{X}_{-t}, \Xi_t, \boldsymbol{\varepsilon}_t\}$, where $\boldsymbol{\varepsilon}_t$ are policy news shocks that we formally introduce below.

To ease future notation we collect all parameters of the general model (6) in

$$\theta = \{\mathcal{A}_{yy}, \mathcal{A}_{yp}, \mathcal{B}_{y\xi}\} \qquad \text{and} \qquad \phi = \{\mathcal{A}_{pp}, \mathcal{A}_{py}, \mathcal{B}_{p\xi}, \mathcal{B}_{p\varepsilon}\}\ . \tag{8}$$

The parameters θ describe the environment that the policy maker faces and ϕ is the reaction function of the policy maker. A critical feature of several of the methods below is that the actions of the policy maker, i.e. ϕ, do not directly influence θ. In other words, the policy maker takes the environment θ as given.

The model (6) is general and accommodates a large class of structural models found in the literature, not only standard New-Keynesian (NK) models (e.g., Smets and Wouters, 2007), but also some modern heterogeneous agents NK models (e.g. Auclert et al., 2021). Further, we note that the model is richer when compared to more standard macro time series models (e.g. conventional structural VARs), which typically only include contemporaneous shocks and have no role for news shocks.

[10]In this work, we focus on stationary environments, in which variables evolve around their steady-state and where the steady-state coincides with the policy targets Y^*. This excludes drifting policy objectives and cases of systematically too low or too high policy instruments; for instance systematic inflation target misses, cases of hyper-inflation, or cases of unsustainable debt.

3 Equilibrium representations and counterfactual policy paths

We will now discuss a number of useful representations for the structural model (6)-(7) that characterize the equilibrium in terms of sufficient macro statistics —impulse responses and forecasts—. A particularly attractive property of these types of representation is that they allow to characterize the equilibrium allocation under *alternative* policy paths (i.e., under alternative parameter vector ϕ) in terms of sufficient statistics alone.

For any specific model in the general class considered we could formulate primitive conditions on the maps $\mathcal{A}_{..}$ and $\mathcal{B}_{..}$ that ensure the existence of a unique and determinate equilibrium. However, since we will generally not be interested in distinguishing among models in the class, we will simply impose a high level assumption on the existence of a *baseline policy rule* vector ϕ^0:

Assumption 1. *There exists a baseline reaction function ϕ^0 under which $\mathcal{A}^0_{pp}$ and $\mathcal{A}_{yy} - \mathcal{A}_{yp}(\mathcal{A}^0_{pp})^{-1}\mathcal{A}^0_{py}$ are invertible maps, i.e. ϕ^0 leads to a unique and determinate equilibrium.*

While the baseline rule could be any rule ensuring a unique equilibrium, in practice it is helpful to think of the baseline rule as a rule that was in place in the recent past, such a sample of data was generated under that baseline rule. This will be necessary to estimate the sufficient macro statistics. In fact, the existence of the baseline rule ensures that we can define impulse responses and forecasts given this rule:

Lemma 1. *Given the generic model* (6)-(7)*, under the policy choice ϕ^0 that satisfies Assumption 1, we have*

$$\begin{array}{rcl} \mathbb{E}_t\mathbf{Y}^0_t & = & \Gamma^0_y\mathbf{S}_t + \mathcal{R}^0_y\boldsymbol{\varepsilon}_t \\ \mathbb{E}_t\mathbf{P}^0_t & = & \Gamma^0_p\mathbf{S}_t + \mathcal{R}^0_p\boldsymbol{\varepsilon}_t \end{array} , \tag{9}$$

where $\mathbf{S}_t = (\Xi'_t, \mathbf{X}'_{-t})'$.

Proof. See Barnichon and Mesters (2023*b*). □

The lemma defines the expected paths for the objectives $\mathbf{Y}^0_t$ and the policy path $\mathbf{P}^0_t$ as a function of the state of the economy $\mathbf{S}_t = (\mathbf{X}'_{-t}, \Xi'_t)'$, which captures initial conditions $\mathbf{X}_{-t}$ and the non-policy news shocks Ξ_t, as well as the policy news shocks $\boldsymbol{\varepsilon}_t$. These expected paths, or oracle forecasts, are conditional on the baseline policy rule ϕ^0 and hence we have indexed the outcomes with a ^{0}.

Simultaneously Lemma 1 defines the impulse responses of $\mathbb{E}_t\mathbf{Y}^0_t$ and $\mathbb{E}_t\mathbf{P}^0_t$ to policy and non-policy shocks. Specifically, we have that $\mathcal{R}^0_j$ captures the impulse responses of $j = y, p$ to policy news shocks at different horizons —from horizon-0 ($\varepsilon_{t,t}$) to any horizon $h > 0$

($\varepsilon_{t,t+h}$)— under the rule ϕ^0. Similarly, Γ_j^0 captures the impulse responses of $j = y, p$ to the state of the economy $\mathbf{S}_t$.

Policy counterfactuals with sufficient macro statistics

Often we are interested in the outcomes under some alternative policy path $\mathbb{E}_t \mathbf{P}_t^1$ that results from an alternative policy rule ϕ^1. Directly mimicking Lemma 1 would lead to impulse responses $\Gamma^1_{..}$ and $\mathcal{R}^1_{..}$ which are defined under the new policy rule ϕ^1. Unfortunately, unless this rule ϕ^1 was used in the past, it is not possible to estimate impulse responses under ϕ^1. To circumvent this we show that there exists a representation of the equilibrium allocation under ϕ^1 in terms of the forecasts and impulse responses *under the baseline rule* ϕ^0. In other words, as long as we can estimate the sufficient statistics under one baseline rule, we can construct any policy counterfactual.

To set this up we first present a useful lemma. Suppose that $\boldsymbol{\delta}_t = (\boldsymbol{\delta}'_{0t}, \boldsymbol{\delta}'_{1t}, \ldots)'$ with $\boldsymbol{\delta}_{jt} \in \mathbb{R}^{M_p}$. We define the modified policy rule

$$\mathcal{A}^0_{pp} \mathbb{E}_t \mathbf{P}_t - \mathcal{A}^0_{py} \mathbb{E}_t \mathbf{Y}_t = \mathbf{X}_{-t} + \mathcal{B}^0_{p\xi} \boldsymbol{\Xi}_t + \mathcal{B}^0_{p\varepsilon} \boldsymbol{\varepsilon}_t + \mathcal{B}^0_{p\varepsilon} \boldsymbol{\delta}_t \ , \tag{10}$$

which adjusts the policy rule (7) under ϕ^0 by $\boldsymbol{\delta}_t$, which is rescaled by $\mathcal{B}^0_{p\varepsilon}$ to ensure that the units of the adjustments are the same as the units of the policy shocks. The following lemma characterizes the equilibrium representation under this adjustment.

Lemma 2. *For any $\boldsymbol{\delta}_t$ such that either $\boldsymbol{\delta}_t$ is deterministic or $\boldsymbol{\delta}_t$ admits a representation $\boldsymbol{\delta}_t = \mathcal{T}_s \mathbf{S}_t + \mathcal{T}_\varepsilon \boldsymbol{\varepsilon}_t$ for arbitrary fixed maps $\mathcal{T}_s, \mathcal{T}_\varepsilon$, given the generic model* (6) *and the modified policy rule* (10) *with ϕ^0 satisfying assumption 1, we have that*

$$\begin{aligned} \mathbb{E}_t \mathbf{Y}_t(\boldsymbol{\delta}_t) &= \mathbb{E}_t \mathbf{Y}_t^0 + \mathcal{R}^0_y \boldsymbol{\delta}_t \\ \mathbb{E}_t \mathbf{P}_t(\boldsymbol{\delta}_t) &= \mathbb{E}_t \mathbf{P}_t^0 + \mathcal{R}^0_p \boldsymbol{\delta}_t \end{aligned}$$

Proof. See Barnichon and Mesters (2023*b*). □

Different policy adjustments can be considered,[11] and Lemma 2 shows that their effects can always be computed from the sufficient statistics defined under the baseline rule ϕ^0. This important lemma can be used in various ways. First and foremost, McKay and Wolf (2023) are interested in the alternative policy rules of the form $\phi^1 = \{\mathcal{A}^1_{pp}, \mathcal{A}^1_{py}, \mathcal{B}^1_{p\xi}, \mathbf{0}\}$ under the assumption that this rule induces a unique equilibrium. To find this counterfactual they

[11] For instance, $\boldsymbol{\delta}_t$ could come from adjustments to the policy rule coefficients, as the $\mathcal{T}$'s represent adjustment to the policy rule reaction coefficients. In particular, $\mathcal{T}_s \mathbf{S}_t$ represents adjustments to the systematic reaction to non-policy shocks $\boldsymbol{\Xi}_t$ or to the state of the economy $\mathbf{X}'_{-t}$ (the initial conditions). Alternatively, $\boldsymbol{\delta}_t$ could be a deterministic adjustment to the rule; an intercept.

show that one needs to choose $\boldsymbol{\delta}_t$ to solve

$$\mathcal{A}^1_{pp}(\mathbb{E}_t\mathbf{P}^0_t + \mathcal{R}^0_p\boldsymbol{\delta}_t) - \mathcal{A}^1_{py}(\mathbb{E}_t\mathbf{Y}^0_t + \mathcal{R}^0_y\boldsymbol{\delta}_t) = \mathbf{X}_{-t} + \mathcal{B}^1_{p\xi}\boldsymbol{\Xi}_t \ .$$

Note that since both the baseline and the counterfactual rule are assumed to induce unique equilibria, Lemma 1 implies that the adjustment can be represented as a function of the predetermined inputs, i.e. there exist maps $\mathcal{T}_s$ and $\mathcal{T}_\varepsilon$ such that $\boldsymbol{\delta}_t = \mathcal{T}_s\mathbf{S}_t + \mathcal{T}_\varepsilon\boldsymbol{\varepsilon}_t$ and Lemma 2 can be used to recover the counterfactuals.

Second, Barnichon and Mesters (2023*b*) choose the optimal $\boldsymbol{\delta}_t$ in order to minimize some loss function. This effectively amounts to treating $\boldsymbol{\delta}_t$ as the choice variable in an optimization problem where the optimal $\boldsymbol{\delta}_t$ subsequently becomes a function of $\mathbf{S}_t$ and $\boldsymbol{\varepsilon}_t$. We discuss this usage in detail in Section 5 below.

Next, we show how lemma 2 allows to characterize the allocation under an alternative policy path $\mathbb{E}_t\mathbf{P}^1_t$ as a function of sufficient macro statistics computed under the baseline rule ϕ_0.

Theorem 1. *The counterfactual macro outcome path under the policy path $\mathbb{E}_t\mathbf{P}^1_t$ can be computed in two steps:*

1. $\boldsymbol{\delta}^{0\to1}_t = (\mathcal{R}^{0\prime}_p\mathcal{R}^0_p)^{-1}\mathcal{R}^{0\prime}_p(\mathbb{E}_t\mathbf{P}^1_t - \mathbb{E}_t\mathbf{P}^0_t)$
2. $\mathbb{E}_t\mathbf{Y}^1_t = \mathbb{E}_t\mathbf{Y}^0_t + \mathcal{R}^0_y\boldsymbol{\delta}^{0\to1}_t$

Proof. Setting $\mathbb{E}_t\mathbf{P}_t(\boldsymbol{\delta}_t) = \mathbb{E}_t\mathbf{P}^1_t$ and using Lemma 2 to solve for $\boldsymbol{\delta}_t$ gives 1. $\boldsymbol{\delta}^{0\to1}_t = (\mathcal{R}^{0\prime}_p\mathcal{R}^0_p)^{-1}\mathcal{R}^{0\prime}_p(\mathbb{E}_t\mathbf{P}^1_t - \mathbb{E}_t\mathbf{P}^0_t)$. Using Lemma 2 again with $\boldsymbol{\delta}_t = \boldsymbol{\delta}^{0\to1}_t$ gives 2. □

The theorem shows how a policy maker who wishes to explore the consequences of a different policy path can use the baseline forecasts and impulse responses to recover the counterfactual. In step 1 the needed adjustment $\boldsymbol{\delta}^{0\to1}_t$ is recovered from the difference between the baseline path $\mathbb{E}_t\mathbf{P}^0_t$ and the desired path $\mathbb{E}_t\mathbf{P}^0_t$. In step 2 the effect of this adjustment on the macro outcomes is computed from the baseline forecast $\mathbb{E}_t\mathbf{Y}^0_t$ and the causal effects of policy on the outcomes $\mathcal{R}^0_y$.

Theorem 1 relies on the identification result of McKay and Wolf (2023) but allows to consider policy counterfactuals in terms of policy path counterfactuals, rather than policy rule counterfactuals. This can be helpful in practice, when policy makers are able to articulate the policy path that they are interested, rather the counterfactual rule that this path implies. Indeed, policy makers need not have an explicit formulation of their desired reaction function.

Theorem 1 has a practical limitation however: it requires the identification of all policy news shocks at different horizons, i.e., the estimation of all the columns of $\mathcal{R}^0_p$ and $\mathcal{R}^0_y$. In practice, this may not be possible. For Theorem 1 this implies that not all columns of $\mathcal{R}_p^0$ and $\mathcal{R}_y^0$ can be recovered from the data. Two general solutions exist: (i) fill, or approximate, the missing columns by extrapolating from the known columns or (ii) compute the best approximating policy path using the available evidence. Strategy (i) is pursued in Caravello, McKay and Wolf (2024), who use a collection of structural macro models, which are fitted using impulse response matching based on the available irfs, to perform the extrapolation. Their approach exploits Lemma 2 in order to avoid specifying a policy rule when doing impulse response matching. Alternatively, de Groot et al. (2021) and Hebden and Winkler (2021) use structural models directly to obtain estimates for the impulse responses to all policy shocks.

Strategy (ii) is simple and can be formalized as follows. Let $\mathcal{R}_{a,p}^0$ and $\mathcal{R}_{a,y}^0$ denote the linear combinations of the columns $\mathcal{R}_p^0$ and $\mathcal{R}_y^0$ that correspond to the policy news shocks $\boldsymbol{\varepsilon}_{a,t} = A\boldsymbol{\varepsilon}_t$ which can be identified.

Corollary 1. *The best linear approximation for the macro outcome path under the policy path $\mathbb{E}_t\mathbf{P}_t^1$ can be computed in two steps:*

1. $\boldsymbol{\delta}_{a,t}^{0\to 1} = (\mathcal{R}_{a,p}^{0\prime}\mathcal{R}_{a,p}^0)^{-1}\mathcal{R}_{a,p}^{0\prime}(\mathbb{E}_t\mathbf{P}_t^1 - \mathbb{E}_t\mathbf{P}_t^0)$
2. $\mathbb{E}_t\mathbf{Y}_t^{1,a} = \mathbb{E}_t\mathbf{Y}_t^0 + \mathcal{R}_{a,y}^0\boldsymbol{\delta}_{a,t}^{0\to 1}$

which is the counterfactual under $\mathbb{E}_t\mathbf{P}_t^0 + \mathcal{R}_{a,p}^0\boldsymbol{\delta}_{a,t}^{0\to 1}$.

The result shows that given only $\mathcal{R}_{a,p}^0$ and $\mathcal{R}_{a,y}^0$ the researcher can only compute the counterfactual under $\mathbb{E}_t\mathbf{P}_t^0 + \mathcal{R}_{a,p}^0\boldsymbol{\delta}_{a,t}^{0\to 1}$, which is the best approximation to $\mathbb{E}_t\mathbf{P}_t^1$ for which the counterfactual can be identified.

4 Inference on counterfactual policy paths

We discuss the estimation of the impulse responses and the oracle forecasts under the baseline policy rule. Given these estimates and their distribution we provide an algorithm for evaluating counterfactual policy paths.

4.1 Estimating impulse responses

We discuss the estimation of impulse responses using aggregate macro economic time series. Theorem 1 reveals that the impulse responses to the policy news shocks, $\mathcal{R}_p^0$ and $\mathcal{R}_y^0$, are of main interest.[12] Following the discussion above, we must often satisfy ourselves with

[12]The Γ's can also be of interest, but for evaluating specific policy counterfactuals (e.g. McKay and Wolf, 2023; Barnichon and Mesters, 2023*a*) or for exploring overall policy performance Barnichon and Mesters (2023*a*) as we will discuss in the next section.

identifying some subsets of these shocks on a subset of the outcome variables. To set this up, let $\mathbb{E}_t\mathbf{D}_t^0 = \mathbb{E}_t(y_t^{0\prime}, \ldots, y_{t+H}^{0\prime}, p_t^{0\prime}, \ldots, p_{t+H}^{0\prime})$ for some finite horizon H, and let $\boldsymbol{\varepsilon}_{a,t}$ denote the subset of policy news shocks that can be identified.

By selecting the appropriate rows from Lemma 1 we obtain

$$\mathbf{D}_s^0 = \mathcal{R}_{a,H}^0 \boldsymbol{\varepsilon}_{a,s} + \mathbf{U}_s \ , \qquad s \in \mathfrak{T} \ . \tag{11}$$

where $\mathcal{R}_{a,H}^0$ is a finite dimensional matrix that includes the entries of the maps $\mathcal{R}_{a,y}^0, \mathcal{R}_{a,p}^0$ implied by our choices for $\mathbf{D}_t$ and $\boldsymbol{\varepsilon}_{a,t}$. The sample is denoted by the set $\mathfrak{T}$, nothing that for time-t policy evaluation often $\mathfrak{T} = \{t_0, \ldots, t-1\}$ will be used whereas for term policy evaluation often the sample is taken as the period over which the policy maker was in office. A key assumption is that over the sampling period the policy rule ϕ^0 was used. The error term $\mathbf{U}_s$ includes all shocks that are not included in $\boldsymbol{\eta}_s$ as well as the future errors $\mathbf{D}_s^0 - \mathbb{E}_s\mathbf{D}_s^0$. By construction, since all structural shocks are assumed to be uncorrelated we have that $\mathbb{E}(\boldsymbol{\varepsilon}_{a,s}\mathbf{U}_s') = 0$.

Equations (11) can be viewed as a set of stacked local projections (e.g. Jordà, 2005). The key difficulty for estimating $\mathcal{R}_{a,H}^0$ is that $\boldsymbol{\varepsilon}_{a,s}$ is not observed and therefore an identification strategy is needed. Prominent examples include using zero-, long-run, or inequality restrictions (e.g. Sims, 1980; Blanchard and Quah, 1989; Faust, 1998; Uhlig, 2005), or by using past exogenous variations as instrumental variables (e.g. Mertens and Ravn, 2013; Stock and Watson, 2018). At the end, pending on preference, any of the identification strategies can be used and often multiple will be needed to identify all shocks of interest, see Ramey (2016) for a broader discussion. After the shocks have been identified conventional econometric methods can be used for estimation and inference.

4.2 Approximating oracle forecasts

The oracle forecasts $\mathbb{E}_t\mathbf{P}_t^0$, $\mathbb{E}_t\mathbf{Y}_t^0$ are defined in Lemma 1 in terms of $\mathbf{S}_t = (\mathbf{X}_{-t}', \Xi_t')'$ and the policy news shocks $\boldsymbol{\varepsilon}_t$. We will discuss two scenarios: (i) the researcher directly downloads the forecasts or (ii) the oracle forecasts need to be approximated by the researcher.

Downloading forecasts

The simplest yet not always feasible way in which a researcher can obtain a baseline forecast is to use the forecasts that are provided by the policy maker. Indeed, several macro policy makers make their forecasts for the policy objectives publicly available and these can then be directly used. Besides using the policy maker's forecasts, the researcher could use also use professional forecasts, such as those from the Survey of Professional Forecasters (SPF) or the Blue Chip forecasts.

Approximating forecasts

Next, we discuss some econometric methods that can be used to approximate the oracle forecasts $\mathbb{E}_t\mathbf{D}_t^0 = \mathbb{E}_t(y_t^{0'}, \ldots, y_{t+H}^{0'}, p_t^{0'}, \ldots, p_{t+H}^{0'})$. We can use Lemma 1 to define the equilibrium representation

$$\mathbb{E}_t\mathbf{D}_t^0 = \Gamma_d^0\mathbf{S}_t + \mathcal{R}_d^0\boldsymbol{\varepsilon}_t^0 \ , \tag{12}$$

where Γ_d^0 and $\mathcal{R}_d^0$ collect the needed rows from Γ_y^0, Γ_p^0 and $\mathcal{R}_y^0, \mathcal{R}_p^0$ in order to correctly define $\mathbb{E}_t\mathbf{D}_t^0$ using Lemma 1.

To approximate $\mathbb{E}_t\mathbf{D}_t^0$ we generally need to approximate the state of the economy $\mathbf{S}_t$ and the policy shocks $\boldsymbol{\varepsilon}_t$. In general, we postulate that the researcher approximates these terms by the (possibly large) vector of time-t observable variables $\mathbf{Z}_t$. Note that $\mathbf{Z}_t$ may include (a part of) the expected policy path $\mathbb{E}_t\mathbf{P}_t^0$ when it is observed to the researcher.

The best linear prediction for $\mathbf{D}_t^0$ in terms of $\mathbf{Z}_t$ can be obtained from the forecasting model over the periods $s \in \mathfrak{T}$.

$$\mathbf{D}_s^0 = \mathbf{B}^0\mathbf{Z}_s + \mathbf{V}_s \ , \qquad s \in \mathfrak{T} \ , \tag{13}$$

where $\mathbf{V}_s$ includes the future error $\mathbf{D}_s^0 - \mathbb{E}_t\mathbf{D}_s^0$ as well as the approximation error that stems from replacing $(\mathbf{S}_s, \boldsymbol{\varepsilon}_s)$ by $\mathbf{Z}_s$. The matrix $\mathbf{B}^0$ is defined such that it includes the best linear prediction coefficients, i.e. the ones that minimize the mean-squared-error, and the error $\mathbf{V}_t$ is orthogonal to $\mathbf{Z}_t$ by construction. In contrast, $\mathbf{V}_t$ is not orthogonal to the total time-t information set $\mathcal{F}_t$, such condition would require perfectly observing the state of the economy which seems a major assumption.[13]

Based on model (13) we can estimate the matrix $\mathbf{B}^0$. This matrix may be structured, e.g. sparse, banded etc, and different estimation methods can allow for shrinkage and penalization to improve the model fit. As examples we can think of: (i) penalized regression methods such as Lasso, Ridge and so on, see Kock, Medeiros and Vasconcelos (2020) for implementation details for time series regressions, (ii) a factor augmented regression where $\mathbf{Z}_s$ form a set of common factors and standard regression methods are used to estimate $\mathbf{B}^0$ (e.g. Stock and Watson, 2002; Bai and Ng, 2006), or (iii) the usage of large (Bayesian) vector autoregressive models (e.g. Banbura, Giannone and Reichlin, 2010).

In general, we denote the estimated model parameters by $\widehat{\mathbf{B}}^0$. The resulting forecasts for

[13] As such we note that $\mathbf{B}^0$ does not have any causal interpretation. Indeed, in contrast to the impulse responses $\Gamma_{..}^0$ and $\mathcal{R}_{..}^0$, these coefficients merely capture the correlation between the observable predictors and the outcome variables of interest.

time period t are given by

$$\widehat{\mathbf{D}}_t^0 = \widehat{\mathbf{B}}^0 \mathbf{Z}_t \ . \tag{14}$$

Clearly the objective is to try to make $\widehat{\mathbf{D}}_t^0$ as close as possible to $\mathbb{E}_t \mathbf{D}_t^0$. At the same time, it is well known that macro forecasting is hard and in practice mistakes will be made.

4.3 Uncertainty

To evaluate policy decisions we generally need the joint distribution of the sufficient statistics. Since, we allow for different forecasting and impulse response estimators we do not give a detailed treatment for any specific choices. Instead we provide a high level overview for how such joint uncertainty measures can be constructed.

We start by recalling our generic forecasting and impulse response equations:

$$\mathbf{D}_s^0 = \mathbf{B}^0 \mathbf{Z}_s + \mathbf{V}_s \qquad \text{and} \qquad \mathbf{D}_s^0 = \mathcal{R}_{a,H}^0 \boldsymbol{\varepsilon}_{a,s} + \mathbf{U}_s \qquad \text{for all} \quad s \in \mathfrak{T} \ .$$

We are interested in the joint uncertainty around $\widehat{\mathbf{D}}_t^0 - \mathbb{E}_t \mathbf{D}_t^0$ — the forecast mis-specification error — and $\widehat{\mathcal{R}}_{a,H}^0 - \mathcal{R}_{a,H}^0$ — the impulse response estimation error. Note that the impulse response estimates $\widehat{\mathcal{R}}_{a,H}^0$ can correspond to OLS, IV or any other desired estimates. To get at the joint uncertainty it is generally useful to recall from (14) that $\widehat{\mathbf{D}}_t = \widehat{B}^0 \mathbf{Z}_t$. This shows that we first need to obtain the joint distribution of the parameter estimates

$$\begin{bmatrix} \widehat{\mathbf{B}}^0 - \mathbf{B}^0 \\ \widehat{\mathcal{R}}_{a,H}^0 - \mathcal{R}_{a,H}^0 \end{bmatrix} \overset{a}{\sim} \widehat{G} \ ,$$

which are generally correlated as $\mathbf{V}_s$ and $\mathbf{U}_s$ overlap in terms of the structural shocks that they contain. The approximating distribution $\widehat{G}$ can be obtained by asymptotic approximations, bootstrap, or Bayesian posteriors. We take no stance on which method should be used.

Second, from $\widehat{\mathbf{D}}_t^0 - \mathbb{E}_t \mathbf{D}_t^0 = (\widehat{\mathbf{B}}^0 - \mathbf{B}^0)\mathbf{Z}_s - \mathbb{E}_t \mathbf{V}_t$, we find that we need to obtain the distribution of $\mathbb{E}_t \mathbf{V}_t$, which is the part of the forecasting model that could have been predicted by the information set $\mathcal{F}_t$, but the researcher did not measure these variables and as such this component ended up in the error term. In other words, this is the model misspecification distribution.

In general, estimating the distribution of $\mathbb{E}_t \mathbf{V}_t$ is hard, and often we will upper bound this quantity by an estimate for the variance of $\mathbf{V}_t$ in combination with a normality assumption (Scheffe, 1953). The variance can be estimated using the mean-squared forecast error variance, see (Stock and Watson, 2019, Section 15.5) for different strategies. In general, the variance of the forecast errors will upper-bound the variance of model uncertainty, because forecast errors mix two sources of uncertainty: (i) model uncertainty *and* (ii) future uncertainty. The latter is typically not needed when evaluating policy decisions as it is outside of the control of the policy maker.

In many settings these inputs can be combined to obtain the joint distribution

$$\begin{pmatrix} \widehat{\mathbf{D}}_t^0 - \mathbb{E}_t \mathbf{D}_t^0 \\ \widehat{\mathcal{R}}_{a,H}^0 - \mathcal{R}_{a,H}^0 \end{pmatrix} \overset{a}{\sim} \widehat{F} \ . \tag{15}$$

The distribution $\widehat{F}$ will be important in our work as this is the distribution from which we will simulate to compute the distribution of the policy evaluation statistics.

4.4 Simulating counterfactual policies

We combine the ingredients discussed above and provide an algorithm for approximating policy counterfactuals. Specifically, given the alternative policy path $\mathbb{E}_t \mathbf{P}_t^1$ we use Corollary 1, or Theorem 1, to learn the counterfactual macro outcomes from the sufficient statistics under ϕ^0: the forecasts and impulse responses. When all impulse responses to policy shocks are recovered the algorithm naturally provides a way to conduct inference on exact policy counterfactuals.

Counterfactual computation

0 Obtain the estimates $\widehat{\mathcal{R}}_{a,y}^0, \widehat{\mathcal{R}}_{a,p}^0$, the forecasts $\widehat{\mathbf{Y}}_t, \widehat{\mathbf{P}}_t$ and the distribution $\widehat{F}$

1 Compute by simulation

$$\begin{aligned} \boldsymbol{\delta}_{a,t}^j &= (\mathcal{R}_{a,p}^{j'} \mathcal{R}_{a,p}^j)^{-1} \mathcal{R}_{a,p}^{j'} (\mathbb{E}_t \mathbf{P}_t^1 - \widehat{\mathbf{P}}_t^j) \\ \widehat{\mathbf{Y}}_t^{j,a} &= \widehat{\mathbf{Y}}_t^j + \mathcal{R}_{a,y}^j \boldsymbol{\delta}_{a,t}^j \end{aligned}$$

where the impulse responses $(\mathcal{R}_{a,p}^j, \mathcal{R}_{a,y}^j)$ and forecasts $(\widehat{\mathbf{P}}_t^j, \widehat{\mathbf{Y}}_t^j)$ are simulated from $\widehat{F}$ for $j = 1, \ldots, S_d$.

2 Report the mean counterfactual paths together with the confidence bands obtained from the simulated distributions.

5 Time-t policy evaluation

In this section we show how the representation results from Section 3 can be used to characterize optimal policy paths and the distance to minimum loss at any given point in time. Moreover, we show how to correct such statistics for dynamic inconsistency stemming from decisions in previous period.

We consider a general quadratic loss function

$$\mathcal{L}_t = \frac{1}{2}\mathbb{E}_t \mathbf{Y}_t' \mathcal{W} \mathbf{Y}_t \ , \tag{16}$$

where $\mathcal{W}$ is a diagonal weighting matrix that allows to place more or less importance on different variables and horizons.[14]

For the general loss function (16) and underlying model (6) we define the optimal allocation as the paths $\mathbb{E}_t\mathbf{Y}_t$ and $\mathbb{E}_t\mathbf{P}_t$ that minimize the loss, i.e.,

$$\min_{\mathbf{Y}_t,\mathbf{P}_t} \mathcal{L}_t \qquad \text{s.t.} \qquad (6) \ . \tag{17}$$

We often refer to the problem (17) as the planner's problem and denote the solution(s) to this problem for the policy path by $\mathbb{E}_t\mathbf{P}_t^{\text{opt}}$ and we denote by $\mathcal{L}_t^{\text{opt}}$ the minimum loss that can be achieved under $\mathbb{E}_t\mathbf{P}_t^{\text{opt}}$.

For clarity of exposition, we make the following simplifying assumption.

Assumption 2. *The optimal policy* $\mathbb{E}_t\mathbf{P}_t^{\text{opt}}$ *is unique.*

The assumption is not essential, and our results continue to hold when replacing $\mathbb{E}_t\mathbf{P}_t^{\text{opt}}$ with a set of optimal policies for which each element of the set solves (17). However, while there could be interesting discriminating aspects among different optimal policies, we retain the uniqueness assumption to avoid notational clutter.

5.1 Optimal policy perturbations

To compute or approximate the optimal policy using sufficient statistics we make use of Lemma 2 which shows that adjusting the policy rule by $\boldsymbol{\delta}_t$ changes the equilibrium outcome to $\mathbb{E}_t\mathbf{Y}_t(\boldsymbol{\delta}_t) = \mathbb{E}_t\mathbf{Y}_t^0 + \mathcal{R}_y^0\boldsymbol{\delta}_t$. To use this result let $\boldsymbol{\delta}_{a,t}$ correspond to the subset of $\boldsymbol{\delta}_t$ for which the corresponding policy shocks can be identified, i.e. $\mathcal{R}_y^0\boldsymbol{\delta}_t = \mathcal{R}_{a,y}^0\boldsymbol{\delta}_{a,t} + \mathcal{R}_{-a,y}^0\boldsymbol{\delta}_{-a,t}$, where the subset of impulse responses $\mathcal{R}_{a,y}^0$ can be identified by the researcher. A special case arises when all shocks can be identified and then we consider $\boldsymbol{\delta}_{a,t} = \boldsymbol{\delta}_t$.

We compute the $\boldsymbol{\delta}_{a,t}$-adjustment that minimizes the loss function. We call this specific $\boldsymbol{\delta}_{a,t}$ the (subset) *Optimal Policy Perturbation* (OPP). Specifically, the idea of the OPP is to

[14] We do not take a stand on the origins of the loss function. As such $\mathcal{L}_t$ can be any desired loss function that the policy maker or researcher wants to minimize. This allows for micro founded, e.g. welfare maximizing, loss functions, but also allows for loss functions that simply correspond to mandates imposed on policy makers. For instance, Bernanke (2015) argued that the Fed should consider inflation and unemployment as its target variables and place equal weight on both objectives over a median term horizon, e.g. five years.

find the "best" adjustment $\boldsymbol{\delta}_{a,t}$ to the baseline rule ϕ^0 in order to minimize the loss, that is

$$\boldsymbol{\delta}_{a,t}^* = \underset{\boldsymbol{\delta}_{a,t}}{\operatorname{argmin}} \ \mathcal{L}_t(\boldsymbol{\delta}_t) \qquad \text{s.t.} \qquad \mathbb{E}_t\mathbf{Y}_t(\boldsymbol{\delta}_t) = \mathbb{E}_t\mathbf{Y}_t^0 + \mathcal{R}_{a,y}^0\boldsymbol{\delta}_{a,t} + \mathcal{R}_{-a,y}^0\boldsymbol{\delta}_{-a,t} \ , \tag{18}$$

where $\mathcal{L}_t(\boldsymbol{\delta}_t) = \frac{1}{2}\mathbb{E}_t\mathbf{Y}_t(\boldsymbol{\delta}_t)'\mathcal{W}\mathbf{Y}_t(\boldsymbol{\delta}_t)$ is the loss function as a function of $\boldsymbol{\delta}_t$. It is easy to see that this adjusted policy problem is linear-quadratic and hence it has a closed form solution given by[15]

$$\boldsymbol{\delta}_{a,t}^* = -(\mathcal{R}_{a,y}^{0'}\mathcal{W}\mathcal{R}_{a,y}^0)^{-1}\mathcal{R}_{a,y}^{0'}\mathcal{W}\mathbb{E}_t\mathbf{Y}_t^0 \ . \tag{19}$$

We can now state the key properties of the OPP, see also Barnichon and Mesters (2023*b*).

Proposition 1. *Given the generic model* (6) *and the augmented policy rule* (10), ϕ^0 *implying a unique equilibrium, we have under Assumption 2 if all policy shocks can be identified, i.e.* $\boldsymbol{\delta}_{a,t}^* = \boldsymbol{\delta}_t^*$, *that*

1. $\mathbb{E}_t\mathbf{P}_t^0 = \mathbb{E}_t\mathbf{P}_t^{\text{opt}} \qquad \Longleftrightarrow \qquad \boldsymbol{\delta}_t^* = \mathbf{0}$
2. $\mathbb{E}_t\mathbf{P}_t^{\text{opt}} = \mathbb{E}_t\mathbf{P}_t^0 + \mathcal{R}_p^0\boldsymbol{\delta}_t^*$.

In contrast, if only a strict subset of policy shocks can be identified

3. $\boldsymbol{\delta}_{a,t}^* \neq 0 \qquad \Longrightarrow \qquad \mathbb{E}_t\mathbf{P}_t^0 \neq \mathbb{E}_t\mathbf{P}_t^{\text{opt}}$
4. $\mathcal{L}_t(\boldsymbol{\delta}_{a,t}^*, \mathbf{0}) \leq \mathcal{L}_t(\mathbf{0}, \mathbf{0})$, *i.e. the adjusted path* $\mathbb{E}_t\mathbf{P}_t^0 + \mathcal{R}_{a,p}^0\boldsymbol{\delta}_{a,t}^*$ *implies a lower loss than the initial path* $\mathbb{E}_t\mathbf{P}_t^0$.

The first and second part consider the case where all policy shocks can be identified. Here we have that if and only if the OPP is zero the policy of interest is equal to the optimal policy. From that property, we can *evaluate* policy decisions: if the OPP is non-zero, we will conclude that the policy path $\mathbb{E}_t\mathbf{P}_t^0$ is not optimal. Second, we can use the OPP to *construct* the optimal policy path $\mathbb{E}_t\mathbf{P}_t^{\text{opt}}$ from some arbitrary baseline policy choice that implies a unique equilibrium.

The third and fourth parts of the proposition consider the case where a strict subset of policy shocks can be identified. Here it holds that if the OPP statistic $\boldsymbol{\delta}_{a,t}^*$ is non-zero the policy $\mathbb{E}_t\mathbf{P}_t^0$ is non-optimal. Moreover, adjusting the baseline policy with the OPP will improve the baseline policy path, though it will generally not give the optimal path $\mathbb{E}_t\mathbf{P}_t^{\text{opt}}$. In other words, the OPP allows to compute the best policy path given the sufficient statistics available. Barnichon and Mesters (2023*b*) provide more discussion regarding the properties of the OPP statistics and the associated adjustments.

[15] It is worth pointing out that throughout we assume that the inverse $(\mathcal{R}_{a,y}^{0'}\mathcal{W}\mathcal{R}_{a,y}^0)^{-1}$ exists. If this is not the case this implies that the effects of the policy instruments are linearly dependent and we can remove one of the instruments from the analysis and simply proceed with the reduced set of instruments for which the invertibility requirement holds.

Correcting for dynamic inconsistency

So far we have considered the problem of a policy maker making a one time decision about the policy path given the time-t information set. This ignores that in most macro policy settings policy decisions are made repeatedly. As is well known, such sequential decision making process creates the possibility of dynamic inconsistency: a policy path that is optimal as of time $t-1$ may not be optimal viewed from a time decision problem as of time t (Kydland and Prescott, 1977). To adjust for this Barnichon and Mesters (2023*b*) introduce a simple correction to the OPP statistic that eliminates dynamic inconsistency.

Specifically, a *time consistent* OPP statistic can be defined as

$$\boldsymbol{\delta}^{\tau *}_{a,t} = \boldsymbol{\delta}^{*}_{a,t} + \Delta \mathcal{D}^0_a \mathbb{E}_{t-1} \mathbf{Y}^0_{t-1} \ , \tag{20}$$

where the original OPP is adjusted with a "time inconsistency correction factor" given by $\Delta \mathcal{D}^0 \mathbb{E}_{t-1} \mathbf{Y}^0_{t-1}$ where $\Delta \mathcal{D}^0_a = \left[\mathcal{D}^0_{a,1} - \mathbf{0}, \mathcal{D}^0_{a,2} - \mathcal{D}^0_{a,1}, ...\right]$ is a "pseudo-difference" map with $\mathcal{D}^0_{a,i}$ the ith $d_a \times M_y$ block of $\mathcal{D}^0_a = -(\mathcal{R}^{0\prime}_{a,y} \mathcal{W} \mathcal{R}^0_{a,y})^{-1} \mathcal{R}^{0\prime}_{a,y} \mathcal{W}$, i.e. $\mathcal{D}^0_a = [\mathcal{D}^0_{a,1}, \mathcal{D}^0_{a,2}, \ldots]$. Importantly, the correction factor is again entirely determined by our two sufficient statistics, so that no extra information is necessary to implement a time-consistent OPP.

Intuitively, the correction factor removes any updates in the original OPP that stem from shifting preferences between time $t-1$ and time t. Indeed the difference map captures the difference in weights placed on the $t-1$ objectives $\mathbb{E}_{t-1} \mathbf{Y}^0_{t-1}$ when considering the $\mathcal{L}_{t-1}$ loss and the $\mathcal{L}_t$. A time consistent policies are only allowed to change because of (i) previous mistakes and (ii) changes in the information set; indeed it is easy to show that

$$\boldsymbol{\delta}^{\tau *}_{a,t} = \boldsymbol{\delta}^{*}_{a,t-1} + \mathcal{D}^0_a \Delta \mathbb{E}_t \mathbf{Y}^0_t \ ,$$

which writes the time consistent OPP as a function of the past OPP (previous mistakes) and the information update $\Delta \mathbb{E}_t \mathbf{Y}^0_t = \mathbb{E}_t \mathbf{Y}^0_t - \mathbb{E}_{t-1} \mathbf{Y}^0_t$. Clearly, for any sequence of periods such corrections can be repeatedly applied to ensure that no dynamic inconsistencies arise among any periods.

It is useful to note that the time-consistent OPP does not lead us to the optimal policy as it was defined in (17). Instead, the planner's problem that has $\boldsymbol{\delta}^{\tau *}_{a,t}$ as optimal adjustment includes a time-consistency restriction:

$$\min_{\mathbf{Y}_t, \mathbf{P}_t} \mathcal{L}_t \qquad \text{s.t.} \qquad (6) \quad \text{and} \quad \mathcal{R}^{0\prime}_y \mathcal{W} \mathbb{E}_{t-1} \mathbf{Y}_t - \mathcal{R}^{0\prime}_y \mathcal{W} \mathbb{E}_{t-1} \mathbf{Y}^0_{t-1} = 0 \ . \tag{21}$$

The constraint imposes that the first order conditions of optimization problem at time t evaluated given the time $t-1$ information (i.e. $\mathcal{R}^{0\prime}_y \mathcal{W} \mathbb{E}_{t-1} \mathbf{Y}_t$) are set equal to the first order conditions from the time $t-1$ policy problem $\mathcal{R}^{0\prime}_y \mathcal{W} \mathbb{E}_{t-1} \mathbf{Y}^0_{t-1}$. Respecting this constraint imposes that all changes in the OPPs between times $t-1$ and t are due to changes in the information set, i.e. moving from $\mathbb{E}_{t-1}$ to $\mathbb{E}_t$. We denote the minimal time consistent loss as defined by (21) by $\mathcal{L}_t^{\tau,\text{opt}}$.

5.2 Time-t distance to minimum loss

The OPP statistic tells us how far the policy maker is from the optimal policy. Clearly, this is one possible metric for evaluating and comparing policy makers. However, we often want to evaluate policy makers or policy institutions based on the loss that could have been avoided by choosing a more optimal policy action. For this we define the Distance to Minimum Loss statistic for time t (DML-t) as

$$\Delta_t = \mathcal{L}_t^0 - \mathcal{L}_t^{\text{opt}} \ , \tag{22}$$

where $\mathcal{L}_t^0$ is the loss under the baseline policy choice ϕ^0 and $\mathcal{L}_t^{\text{opt}}$ is the loss under the optimal policy as defined in (17). If the entire optimal policy perturbation can be recovered from the sufficient statistics, i.e. if all policy news shocks can be identified, we can compute the DML-t using

$$\Delta_t = \boldsymbol{\delta}_t^{\tau*\prime} \mathcal{R}_y^{0\prime} \mathcal{W} \mathcal{R}_y^0 \boldsymbol{\delta}_t^{\tau*} \ , \tag{23}$$

which expresses the DML-t in terms of the OPP and impulse responses under the baseline policy choice.

In practice, typically not all policy news shocks can be identified and we instead compute the distance to minimum loss that we can empirical identify. That is

$$\Delta_{a,t} = \mathcal{L}_t^0 - \mathcal{L}_t(\boldsymbol{\delta}_{a,t}^*, 0) \ ,$$

where $\mathcal{L}_t(\boldsymbol{\delta}_t) = \frac{1}{2}\mathbb{E}_t \mathbf{Y}_t(\boldsymbol{\delta}_t)' \mathcal{W} \mathbf{Y}_t(\boldsymbol{\delta}_t)$ is the loss function which is here evaluated at the subset optimal policy choice $\boldsymbol{\delta}_{a,t}^*$. The difference is that $\mathcal{L}_t(\boldsymbol{\delta}_{a,t}^*, 0)$ is not the exact optimal policy, but the best approximation thereof that can be obtained using the empirical evidence. This subset of the distance to minimum loss can be computed from

$$\Delta_{a,t} = \boldsymbol{\delta}_{a,t}^{\tau*\prime} \mathcal{R}_{a,y}^{0\prime} \mathcal{W} \mathcal{R}_{a,y}^0 \boldsymbol{\delta}_{a,t}^{\tau*} \ . \tag{24}$$

In the case where the optimal policy is defined to be time consistent as in (21) we have that

$$\begin{aligned}
\Delta_{a,t}^{\tau} =& \mathcal{L}_t^0 - \mathcal{L}_t^{\tau,\text{opt}} \\
=& -\boldsymbol{\delta}_{a,t}^{\tau*\prime} \mathcal{R}_y^{0\prime} \mathcal{W} \mathcal{R}_y^0 \boldsymbol{\delta}_{a,t}^{\tau*} - 2\boldsymbol{\delta}_{a,t}^{\tau*\prime} \mathcal{R}_y^{0\prime} \mathcal{W} \mathbb{E}_t \mathbf{Y}_t^0 \ ,
\end{aligned}$$

which incorporates the corrections for dynamic inconsistency. As the definition of the time consistent OPP includes the correction we need to take this into account when computing the time consistent DML-t.

Implementation OPP and DML statistics

Next, we formalize the implementation of the OPP and DML statistics. Broadly speaking we can use any of the forecasting and impulse response estimation methods discussed in Section 4 to obtain the approximating distribution $\widehat{F}$ of $(\widehat{\mathcal{R}}^0_{a,y} - \mathcal{R}^0_{a,y}, \widehat{\mathcal{R}}^0_{a,p} - \mathcal{R}^0_{a,p}, \widehat{\mathbf{Y}}_t - \mathbb{E}_t\mathbf{Y}^0_t, \widehat{\mathbf{P}}_t - \mathbb{E}_t\mathbf{P}^0_t)$. Subsequently, we can simulate from this distribution to obtain the distribution of the OPP adjustment and compute the other statistics.

The following algorithm describes the procedure in detail.

OPP and DML computation

0 Obtain the estimates $\widehat{\mathcal{R}}^0_{a,y}, \widehat{\mathcal{R}}^0_{a,p}$, the forecasts $\widehat{\mathbf{Y}}_t, \widehat{\mathbf{P}}_t$ and the distribution $\widehat{F}$

1 Compute for a given matrix $\mathcal{W}$ by simulation

$$
\begin{aligned}
\boldsymbol{\delta}^j_{a,t} &= -(\mathcal{R}^{j\prime}_{a,y}\mathcal{W}\mathcal{R}^j_{a,y})^{-1}\mathcal{R}^{j\prime}_{a,y}\mathcal{W}\widehat{\mathbf{Y}}^j_t \\
\Delta^j_{a,t} &= \boldsymbol{\delta}^{j\prime}_{a,t}\mathcal{R}^{j\prime}_{a,y}\mathcal{W}\mathcal{R}^j_{a,y}\boldsymbol{\delta}^j_{a,t}
\end{aligned}
$$

or

$$
\begin{aligned}
\boldsymbol{\delta}^{\tau,j}_{a,t} &= \boldsymbol{\delta}^j_{a,t} - \Delta\mathcal{D}^j_a\widehat{\mathbf{Y}}^j_{t-1} \\
\Delta^{\tau,j}_{a,t} &= -\boldsymbol{\delta}^{\tau,j\prime}_{a,t}\mathcal{R}^{j\prime}_{a,y}\mathcal{W}\mathcal{R}^j_{a,y}\boldsymbol{\delta}^{\tau,j}_{a,t} - 2\boldsymbol{\delta}^{\tau,j\prime}_{a,t}\mathcal{R}^{j\prime}_{a,y}\mathcal{W}\widehat{\mathbf{Y}}^j_t
\end{aligned}
$$

where the impulse responses $(\mathcal{R}^j_{a,p}, \mathcal{R}^j_{a,y})$ and forecasts $(\widehat{\mathbf{P}}^j_t, \widehat{\mathbf{Y}}^j_t)$ are simulated from $\widehat{F}$ for $j = 1, \ldots, S_d$.

2 For each draw compute the adjusted paths

$$
\widehat{\mathbf{W}}^{a,j}_t = \widehat{\mathbf{W}}^j_t + \mathcal{R}^j_{a,w}\boldsymbol{\delta}^j_{a,t}\ , \qquad \mathbf{W} = \mathbf{P}, \mathbf{Y}
$$

or

$$
\widehat{\mathbf{W}}^{\tau a,j}_t = \widehat{\mathbf{W}}^j_t + \mathcal{R}^j_{a,w}\boldsymbol{\delta}^{\tau,j}_{a,t}\ , \qquad \mathbf{W} = \mathbf{P}, \mathbf{Y}
$$

3 Report the mean OPP statistics and the adjusted policy paths together with the confidence bands obtained from the simulated distributions.

6 Term policy evaluation

We consider the evaluation of a policy maker over her term. The policy maker's problem is to choose a policy rule $\phi = \{\mathcal{A}_{pp}, \mathcal{A}_{py}, \mathcal{B}_{p\xi}, \mathcal{B}_{p\epsilon}\}$ that minimizes the unconditional loss $\mathcal{L}(\phi, \theta) = \mathbb{E}\mathcal{L}_t$. That is choose a policy rule from the set

$$\Phi^{\text{opt}} = \{\phi \in \Phi : \phi \in \underset{\phi}{\text{argmin}}\, \mathcal{L}(\phi, \theta) \quad \text{s.t.} \quad (6) \text{ and } (7)\} \ . \tag{25}$$

Broadly speaking, choosing an optimal rule for an unconditional objective function corresponds to the timeless perspective for optimal policy making, see Woodford (2003) and Giannoni and Woodford (2004) for more discussion.

6.1 Distance to minimum loss

In this context, we measure the policy makers performance by considering the (unconditional) distance to minimum loss

$$\Delta = \mathcal{L}^0 - \mathcal{L}^{\text{opt}} \ , \tag{26}$$

where $\mathcal{L}^0 = \mathcal{L}(\phi^0, \theta)$ and $\mathcal{L}^{\text{opt}} = \mathcal{L}(\phi^{\text{opt}}, \theta)$ with $\phi^{\text{opt}} \in \Phi^{\text{opt}}$. This is the difference in loss that results from the choosing a possibly sub-optimal reaction function ϕ^0. In contrast, the time-t distance to minimum loss in (22) is a function of the time-t information set and mixes policy mistakes $\boldsymbol{\varepsilon}_t$ with a sub-optimal policy rule.

To characterize the distance to optimality in terms of sufficient statistics we apply Lemma 2 with the specific rule adjustment

$$\boldsymbol{\delta}^b_{a,t} = \mathcal{T}_{\xi,b}\boldsymbol{\Xi}_{b,t} + \mathcal{T}_{\varepsilon,a}\boldsymbol{\varepsilon}_{a,t} \ ,$$

where $\mathcal{T}_{\xi,b}$ is an adjustment to the reaction to non-policy shocks $\boldsymbol{\Xi}_{b,t}$ and $\mathcal{T}_{\varepsilon,a}$ is an adjustment to the reaction to policy shocks $\boldsymbol{\varepsilon}_{a,t}$, see footnote 11. We have

$$\mathbb{E}_t\mathbf{Y}_t(\boldsymbol{\delta}^b_{a,t}) = (\Gamma^0_{y,\xi_b} + \mathcal{R}^0_{a,y}\mathcal{T}_{\xi,b})\boldsymbol{\Xi}_{b,t} + (\mathcal{R}^0_{a,y} + \mathcal{R}^0_{a,y}\mathcal{T}_{\varepsilon,a})\boldsymbol{\varepsilon}_{a,t} + N_t \ ,$$

where N_t includes all shocks that are not adjusted by the perturbation[16] and Γ^0_{y,ξ_b} are the impulse responses of $\mathbf{Y}_t$ to shocks $\boldsymbol{\Xi}_{b,t}$. The result shows that the $\mathcal{T}_{ab} = [\mathcal{T}_{\xi,b}, \mathcal{T}_{\varepsilon,a}]$ adjustments to the reaction coefficients change the impulse responses from Γ^0_{y,ξ_b} to $\Gamma^0_{y,\xi_b} + \mathcal{R}^0_{a,y}\mathcal{T}_{\xi,b}$ and from $\mathcal{R}^0_{a,y}$ to $\mathcal{R}^0_{a,y} + \mathcal{R}^0_{a,y}\mathcal{T}_{\varepsilon,a}$. Crucially the adjusted impulse responses are a function of the impulse responses under ϕ^0, i.e., we can compute the new impulse responses using the methods from Section 4.

[16] Formally, $N_t = \Gamma^0_{y,x}\mathbf{X}_{-t} + \Gamma^0_{y,\xi_{-b}}\boldsymbol{\Xi}_{-b,t} + \mathcal{R}^0_{-a,y}\boldsymbol{\varepsilon}_{-a,t}$.

From these "law of motions" for the impulse responses, we can compute the *optimal* $\boldsymbol{\delta}^b_{a,t}$ adjustment to the policy rule, i.e. the $\mathcal{T}_{ab}$ adjustments that minimize the loss function $\mathcal{L}(\phi,\theta)$. When all shocks can be identified, this allows to compute the optimal loss $\mathcal{L}^{\text{opt}}$ and thus the distance to minimum loss Δ. The following proposition summarizes the results.

Proposition 2. *Given the generic model* (6) *and the augmented policy rule* (10)*,* ϕ^0 *implying a unique equilibrium, we have under Assumption 2 if all policy and non-policy shocks can be identified we have that*

1. $\Delta = \Delta_\xi + \Delta_\varepsilon$ *with*

$$\Delta_\xi = \text{Tr}\left(\Gamma^{0\prime}_{y,\xi}\mathcal{W}\mathcal{R}^0_y(\mathcal{R}^{0\prime}_y\mathcal{W}\mathcal{R}^0_y)^{-1}\mathcal{R}^{0\prime}_y\mathcal{W}\Gamma^0_{y,\xi}\right) \quad \textit{and} \quad \Delta_\varepsilon = \text{Tr}\left(\mathcal{R}^{0\prime}_y\mathcal{W}\mathcal{R}^0_y\right)$$

If only a strict subset of policy and non-policy shocks can be identified we have

2. $\Delta_{ab} = \mathcal{L}^0 - \min_{\mathcal{T}_{ab}} \frac{1}{2}\mathbb{E}(\mathbf{Y}_t(\boldsymbol{\delta}^b_{a,t})'\mathcal{W}\mathbf{Y}_t(\boldsymbol{\delta}^b_{a,t})) = \Delta_{\xi,ab} + \Delta_{\varepsilon,aa}$ *with*

$$\Delta_{\xi,ab} = \text{Tr}\left(\Gamma^{0\prime}_{y,\xi_b}\mathcal{W}\mathcal{R}^0_{a,y}(\mathcal{R}^{0\prime}_{a,y}\mathcal{W}\mathcal{R}^0_{a,y})^{-1}\mathcal{R}^{0\prime}_{a,y}\mathcal{W}\Gamma^0_{y,\xi_b}\right) \quad \textit{and} \quad \Delta_{\varepsilon,aa} = \text{Tr}\left(\mathcal{R}^{0\prime}_{a,y}\mathcal{W}\mathcal{R}^0_{a,y}\right) .$$

The first part of proposition 2 makes an important point: a policy maker follows an optimal rule if and only if she responds optimally to each new set of shocks that she faces during her term. This result implies that we can characterize the distance to minimum loss using sufficient macro statistics, and is at the core of the sufficient statistics approach to policy evaluation.

The second part of proposition 2 shows that if only a subset of shocks can be identified we can recover Δ_{ab}; a part of the total distance to minimum loss Δ. A proof is given in Barnichon and Mesters (2023*a*) who also provide bounds on the share of Δ that is captured by Δ_{ab}.

Moreover, the proposition shows that we can decompose the distance to minimum loss into different interpretable components. First, $\Delta_{\xi,ab}$ captures the sub-optimal reaction to the non-policy shocks. We can write

$$\Delta_{\xi,ab} = \sum_{j\in b}\mathcal{T}'_{a,j}\mathcal{R}^{0\prime}_{a,y}\mathcal{W}\mathcal{R}^0_{a,y}\mathcal{T}_{a,j} \quad \text{with} \quad \mathcal{T}_{a,j} = -(\mathcal{R}^{0\prime}_{a,y}\mathcal{W}\mathcal{R}^0_{a,y})^{-1}\mathcal{R}^{0\prime}_{a,y}\mathcal{W}\Gamma^0_{y,j} \ ,$$

which shows that each increment $\mathcal{T}'_{a,j}\mathcal{R}^{0\prime}_{a,y}\mathcal{W}\mathcal{R}^0_{a,y}\mathcal{T}_{a,j}$ captures the loss that is due to the non-optimal response to shocks of type $\boldsymbol{\Xi}_{b_j,t}$ when setting the policy instrument corresponding to the shocks $\boldsymbol{\varepsilon}_{a,t}$. The statistic $\mathcal{T}_{a,j}$ is the Optimal Reaction Function (ORA) statistic that was introduced in Barnichon and Mesters (2023*a*); it captures how the systematic policy response to the shock $\Xi_{b_j,t}$ should be adjusted to minimize the loss function. The other component $\Delta_{\varepsilon,aa}$ captures the loss that could have been avoided by not making policy mistakes.

6.2 Recovering the DML from time-t statistics

The distance to minimum loss can also be computed using the time-t sufficient statistics. For this we define the first difference of the OPP statistic (19) as

$$\boldsymbol{\delta}_{a,t}^{\Delta} = -(\mathcal{R}_{a,y}^{0'}\mathcal{W}\mathcal{R}_{a,y})^{-1}\mathcal{R}_{a,y}^{0'}\mathcal{W}\Delta\mathbb{E}_t\mathbf{Y}_t \quad \text{with} \quad \Delta\mathbb{E}_t\mathbf{Y}_t = \mathbb{E}_t\mathbf{Y}_t - \mathbb{E}_{t-1}\mathbf{Y}_t \ .$$

Intuitively, $\boldsymbol{\delta}_{a,t}^{\Delta}$ captures the sub-optimal response of the policy maker to the forecast revisions.

Corollary 2. *Given the generic model* (6) *with Assumptions 1-2, given the augmented policy rule* (10)*, if all policy shocks can be identified we have that*

1. $\Delta = \mathbb{E}(\boldsymbol{\delta}_t^{\Delta'}\mathcal{R}_y^{0'}\mathcal{W}\mathcal{R}_y^{0}\boldsymbol{\delta}_t^{\Delta})$

If not all policy *shocks can be identified we have*

2. $\Delta_a = \mathbb{E}(\boldsymbol{\delta}_{a,t}^{\Delta'}\mathcal{R}_{a,y}^{0'}\mathcal{W}\mathcal{R}_{a,y}^{0}\boldsymbol{\delta}_{a,t}^{\Delta})$ *where* $\Delta_a = \sum_b \Delta_{ab}$

Proof. Note that

$$\begin{aligned}\Delta &= \mathbb{E}(\Delta\mathbb{E}_t\mathbf{Y}_t'\mathcal{W}\mathcal{R}_y^0(\mathcal{R}_y^{0'}\mathcal{W}\mathcal{R}_y)^{-1}\mathcal{R}_y^{0'}\mathcal{W}\Delta\mathbb{E}_t\mathbf{Y}_t)\\ &= \mathrm{Tr}(\mathcal{W}\mathcal{R}_y^0(\mathcal{R}_y^{0'}\mathcal{W}\mathcal{R}_y)^{-1}\mathcal{R}_y^{0'}\mathcal{W}\mathbb{E}(\Delta\mathbb{E}_t\mathbf{Y}_t\Delta\mathbb{E}_t\mathbf{Y}_t'))\end{aligned}$$

By Lemma 1 we have

$$\Delta\mathbb{E}_t\mathbf{Y}_t = \Gamma_{y,\xi}\boldsymbol{\Xi}_t + \mathcal{R}_y\boldsymbol{\varepsilon}_t \ ,$$

as $\mathbf{X}_{-1}$ is time $t-1$ measurable and hence cancels out. Using that are shocks are normalized to have mean zero and unit variance, we have that

$$\mathbb{E}(\Delta\mathbb{E}_t\mathbf{Y}_t\Delta\mathbb{E}_t\mathbf{Y}_t') = \mathcal{R}_y^{0'}\mathcal{R}_y^{0} + \Gamma_{y,\xi}^{0'}\Gamma_{y,\xi}^{0} \ .$$

Substituting this back into the first expression gives $\Delta = \Delta_\xi + \Delta_\varepsilon$ as they are defined in Proposition 2. The proof of the second part follows using similar steps. □

Note that the subset DML $\Delta_a = \sum_b \Delta_{ab}$ is the total distance to minimum loss *for the policy instruments* a, and it is equal to the sum of the subset DMLs Δ_{ab}, the distance to minimum loss for the a policy instruments' reaction to each non-policy shocks (indexed by b).

Corollary 2 is a new result that links the time-t perspective (Barnichon and Mesters, 2022) with the timeless perspective Barnichon and Mesters (2023*a*). It provides researchers with simple ways for estimating the DML Δ when the OPP statistics have been computed. Specifically, computing the DML simply amounts to taking a weighted sum of squares of the differenced OPP statistics over a policy maker's term. For a policy maker starting in t_0 and ending in $t_0 + J$ this would be

$$\widetilde{\Delta}_a = \frac{1}{J} \sum_{t=t_0}^{t_0+J} \boldsymbol{\delta}_{a,j}^{\Delta\prime} \mathcal{R}_{a,y}^{0\prime} \mathcal{W} \mathcal{R}_{a,y}^{0} \boldsymbol{\delta}_{a,j}^{\Delta} \ ,$$

where the simulation methods of Section 5 can be used to obtain estimates for $\boldsymbol{\delta}_{a,j}^{\Delta}$. This approach avoids the need to identify all non-policy shocks.

7 Monetary policy in the Euro area

The European Central Bank (ECB) has been making policy decisions in the Euro area since the adoption of the euro in 1999. The first twenty years of ECB's history have been extensively reviewed in Hartman and Smets (2018). In particular, Hartman and Smets (2018) evaluate past ECB policy decisions through the lens of estimated Taylor rules capturing the ECB's reaction function. Throughout this section we will revisit some of their findings through the lens of the sufficient statistics approach.

We start by outlining the set-up. First, we consider the Euro area as the unit of analysis. This means that we consider aggregate Euro area variables as the variables in $\mathbf{Y}_t$ and the model that generated these variables is assumed to be of the form (6).[17] The policy instrument that we consider is the short term interest rate i_t, and the expected policy path is $\mathbb{E}_t\mathbf{P}_t = \mathbb{E}_t(i_t, i_{t+1}, \ldots)'$ and the policy shocks are the contemporaneous and news shocks collected in $\boldsymbol{\varepsilon}_t$. This view imply that we only considered policies of the ECB that affect the economy through the expected interest rate path, see also Eberly, Stock and Wright (2020).[18]

[17]The alternative is to use the individual countries as the unit of analysis. See the recent evaluation of Einarsson (2024) based on sufficient macro statistics at the country level.

[18]The ECB policy toolkit is not restricted to the policy rate path, and balance sheet operations have also been used since 2007. We leave the evaluation of such policies for future work.

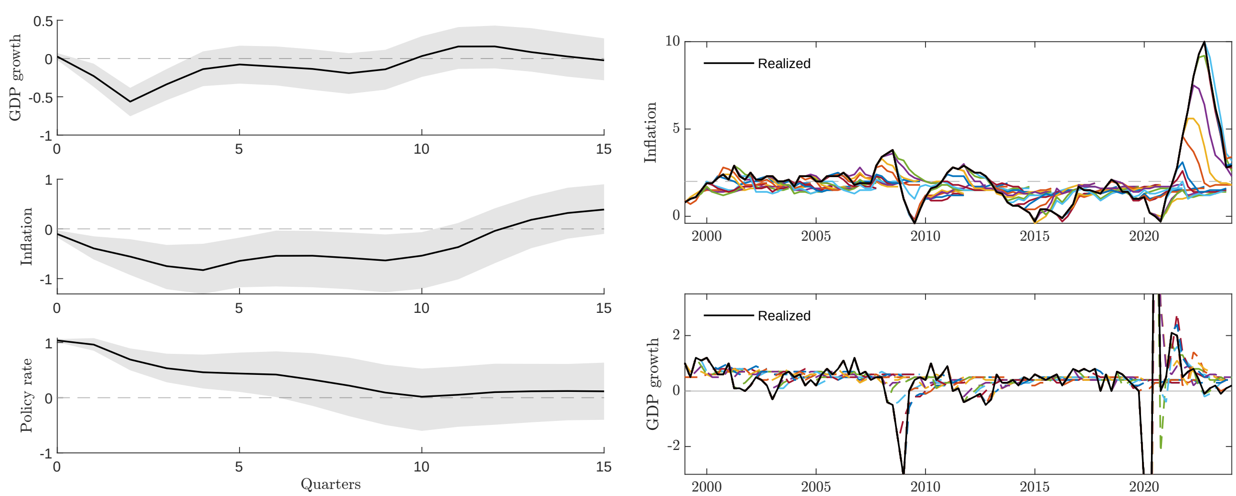
7.1 Estimating the sufficient statistics

Impulse responses to monetary shocks

To evaluate ECB policy decisions, we will rely on the impulse responses to a single policy shock. This implies that we only use one linear combination of the columns of $\mathcal{R}_y^0$ and $\mathcal{R}_p^0$ for evaluation. The impulse responses imply that our policy assessment will be focused on the short-end of the policy path; roughly over the next year. Our evaluation will be silent about the optimality of the medium- to longer-end of the policy path.

To identify the policy shock of interest we consider a mixed frequency Bayesian vector autoregressive model that includes (HICP, year-on-year, monthly), real GDP growth (quarter-on-quarter, quarterly), short term interest rate (EONIA rate extended with the Euro Short Term Rate (€STR, monthly), commodity price index (monthly) and the spread between the short term interest rate and the ten year German yield (monthly). We identify the monetary policy shock of interest by ordering the short term interest rate last and using recursive zero restrictions (e.g. Kilian and Lütkepohl, 2017, Section 8.2).[19]

Figure 1: THE SUFFICIENT MACRO STATISTICS



(a) Impulse responses to policy shock

(b) ECB macroeconomic forecasts

Notes: Inflation is the y-to-y change in the Harmonised Index of Consumer Prices (HICP), and the policy rate is the Euro Overnight Index Average (EONIA).

We estimate the VAR using data from January 2002 until December 2019. We impose a conventional Minnesota prior on the reduced form coefficients (e.g. Canova, 2007) and use $p = 24$ lags to avoid biases from short lag lengths. The results are shown in Figure 1

[19] Alternative identification schemes are possible. For instance, high frequency identified monetary surprises (e.g. Altavilla et al., 2019; Odendahl et al., 2024).

below.[20] We find that the effect on inflation is significantly negative and persistent for at least 10 quarters. In contrast, the effect on real GDP growth fades out after approximately four quarters.

The oracle forecasts

To approximate the oracle forecasts for interest rates, inflation, growth, unemployment and possibly other variables, several routes can be followed. Here we rely on the macro economic projections provided by the ECB. The forecasts are available from 1999 onward and are published four times a year (in March, June, September and December). We include all forecasts up to December 2023. Each forecasts is around 10 quarters into the future with slight variation across the reporting periods.

The forecasts are shown in the right-panel of Figure 1 together with realized inflation and GDP growth. It is easy to visually confirm that indeed the forecasts mean revert quickly, yet at the same time their accuracy is comparable to conventional time series model forecasts (Kontogeorgos and Lambrias, 2019). Unfortunately, the ECB does not provide any measure of model uncertainty, e.g. stemming from parameter estimates or other specification choices, in their publications.

7.2 Time-t ECB Policy evaluation

We evaluate the ECB policy decisions based on the loss function

$$\mathcal{L}_t = \mathbb{E}_t \sum_{j=1}^{H} (\pi_{t+j} - \pi^*)^2 + \lambda (x_{t+j} - x^*)^2 \ ,$$

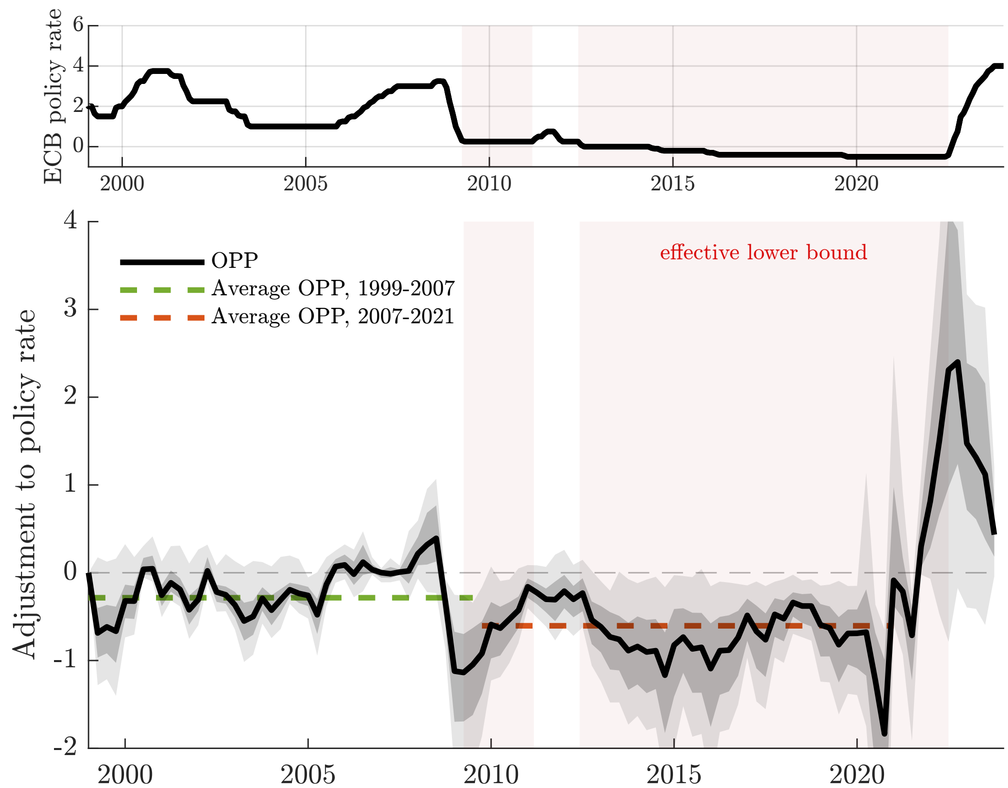
where the inflation target π^* is set to 2% and potential output is set to $x^* = 0.6$, which is the average value of GDP growth over the 1999-2006 period, and the preference parameter $\lambda = 1$ with $h = 16$ quarters. The main findings are robust to reasonable changes in these targets. Using the algorithm described above we compute the distribution of the OPP statistic for each quarter over the 1999-2023 period. The average OPP statistic is shown in Figure 3 together with the 67 and 95% confidence bands reflecting impulse response estimation uncertainty.[21]

With the caveat that our evaluation is restricted to the short-end of the policy path, we find that the ECB interest rate policy has been largely optimal during the early years of

[20] Since the forecasts that we use below are only on a quarterly level we average the resulting impulse responses to the quarterly frequency.

[21] Since we do not have a measure of model uncertainty for the ECB forecasts, the confidence bands are only based on IRF uncertainty.

Figure 2: OPP statistics for Euro area monetary policy



Notes: Top panel: the ECB Deposit Facility Rate. bottom panel: Adjustment to contemporaneous ECB policy rate as implied by the baseline OPP (thick line). Shaded areas report the 67 and 95% confidence bands. The green dashed line depicts the average OPP over 1999-2007, and the red dashed line depicts the average OPP over 2007-2021.

the ECB, though the policy rate was set too slightly too high on average with an average OPP of about $-.25$ ppt. This reflects an inflation rate running persistently below 2 percent over that period. Starting in 2006, with inflation running slightly above 2 percent, the OPP makes a mild case for tightening, before plunging swiftly into negative territories with the dramatic collapse in GDP growth caused by the financial crisis. An interesting finding is that the OPP calls for stronger interest rate cuts in the early stage of the Great Recession, when the zero lower bound was *not yet* binding. In the first meeting of 2009, when the ECB deposit rate was still at 1.65 ppt, the OPP calls for a full 1 percentage point cut, a cut that the ECB will ultimately implement but only progressively and with a 6 months delay. Arguably, this earlier reaction could have attenuated some of the effects of the financial shock. This finding is analog to what was found for the Fed in Barnichon and Mesters (2023*b*). After 2010, the ECB entered a prolonged period (2009-2021) where conventional monetary policy was constrained by the zero/effective lower-bound. Not surprisingly, the average OPP over 2009-2021 is $-.6$ ppt, lower than the $-.25$ppt average over 1999-2008;

this captures the fact that the lower bound on the policy rate did constrain monetary policy in the Euro area. Interestingly, while the constraint on ECB interest policy is about half a percentage .5ppt, and similar to the ZLB constraint on US monetary policy (Barnichon and Mesters, 2023*b*), the duration of the constraint is much longer. While US monetary policy was only restricted over 5 years (2009-2014), Euro area monetary policy was constrained for more than 10 years (2009-2021). While this reflects the effect of European debt crisis that affected southern European countries over 2010-2014, this can also point to a less resilient Euro economy/more sclerotic the labor market (implying slower rebound from troughs) or to a lower value for the natural interest rate r^* in the Euro area than in the US. [22].

During the COVID recovery, inflation surged and the OPP calls for large increases in the policy rate, as much as 2 percentage points in 2022, a time when the ECB stayed put with the policy rate stuck at the lower bound. That said, a number of caveat are important. First, our loss function ignores any type of interest rate smoothing motive, which would penalize large changes in the policy rate. Another caveat is that of forward guidance: in its July 2021 monetary policy decision Press Release, the ECB stated that the "Governing Council expects the key ECB interest rates to remain at their present or lower levels *until it sees inflation reaching two per cent well ahead of the end of its projection horizon and durably for the rest of the projection horizon*". Further, the policy statement added that "This may also imply a transitory period in which inflation is moderately above target". With such a promise in place, it made sense for the ECB to delay its lift-off in the face of above-2 percent inflation. All that said, with headline inflation as high as 10 percent (substantially higher than US headline inflation), the case for a faster monetary reaction is hard to dismiss.

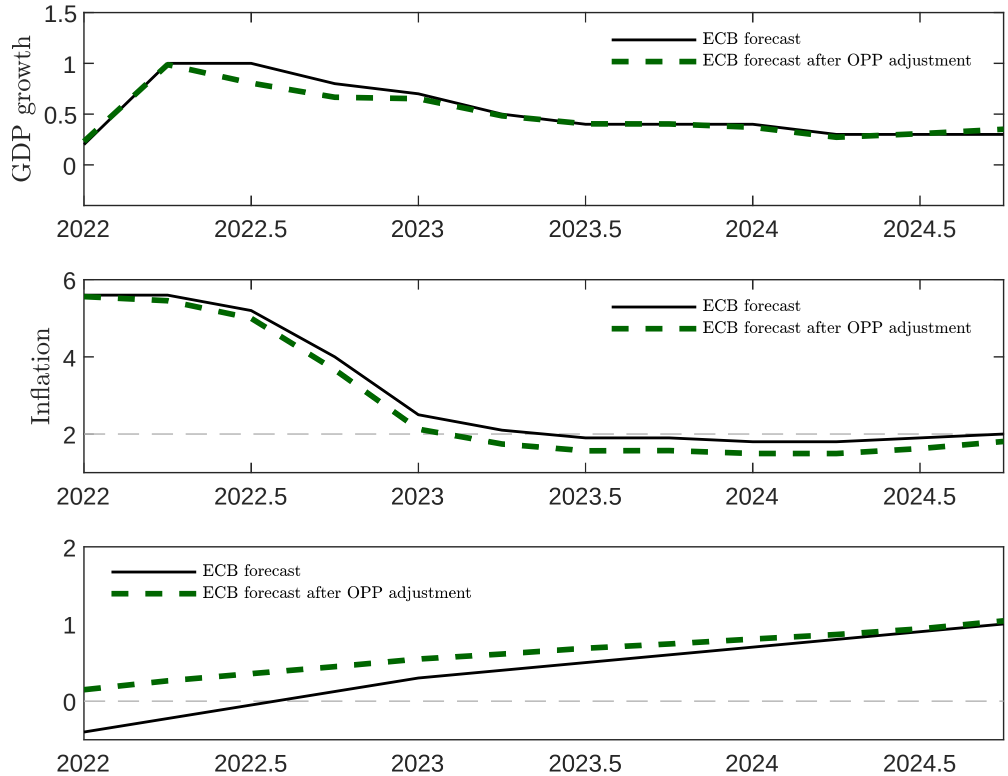
To better illustrate the sub-optimal delayed reaction of the ECB, we zoom in on the first quarter of 2022 policy decision and Figure reports the ECB forecasts as of February 3, 2022, along with the OPP adjusted paths. At the time the short term interest rates were fixed at the zero lower bound and the ECB did not start raising rates until July 2022. By having an earlier lift-off (and raising the policy rate from -0.5 to about $+0.25$, the ECB could have brought down inflation faster, reaching its inflation target about 6 months earlier (in expectation). The cost would have been lower GDP growth in 2022, by about 0.25 ppt.

7.3 Overall (term) ECB policy evaluation

As last exercise, we can evaluate the overall ECB performance over 1999-2023 based on the timeless perspective, and we compute $\Delta_a = \mathbb{E}\delta_{a,t}^{\Delta\prime}\left(\mathcal{R}'\mathcal{W}\mathcal{R}\right)\delta_{a,t}^{\Delta}$. Figure 4 plots the OPP innovation series $\delta_{a,t}^{\Delta}$, and we can clearly see the two main sub-optimal decision dates in ECB's short history: in 2009 when the ECB did not lower interest rates enough in the face

[22]Ceteris paribus, a lower r^* makes the ZLB constraint more likely to bind (e.g., Le Bihan et al., 2019).

Figure 3: OPP ADJUSTED PATHS 2022-Q1



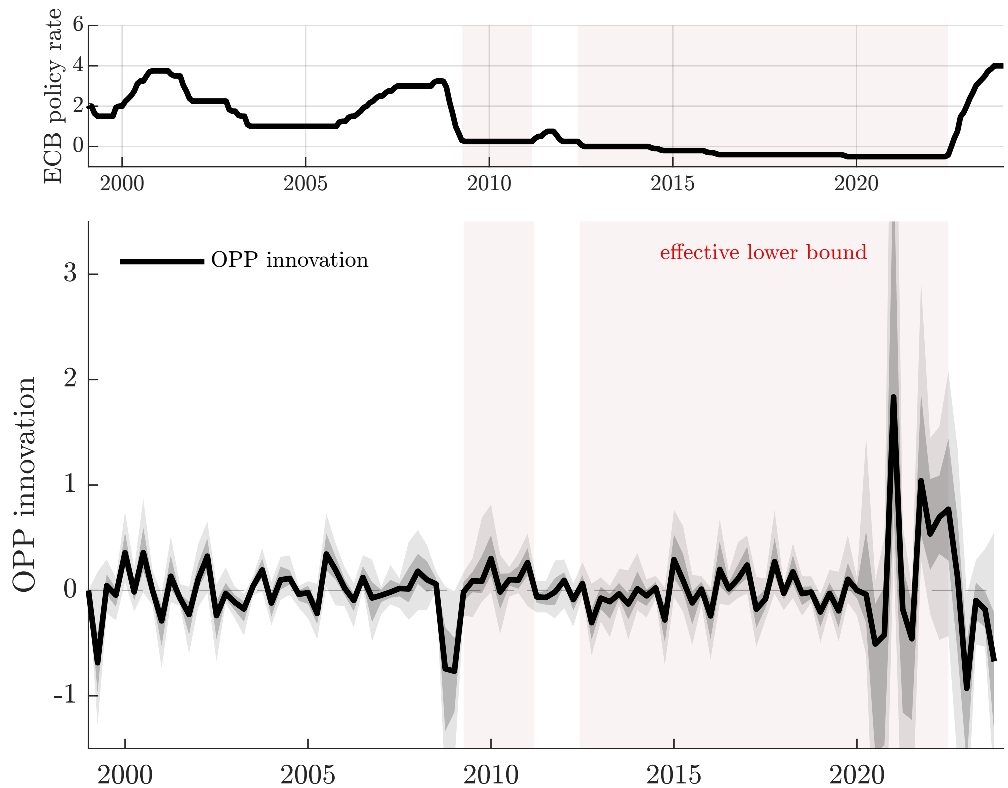
Notes: Plain lines: Expected paths for GDP growth, Inflation (HICP) and the ECB policy rate (Marginal Lending Facility Rate). Dashed green lines: Corresponding expected path after OPP-adjustment of the policy rule.

of deteriorating forecasts and mounting risks to unemployment, and in 2022 when the ECB did not raise interest rates in the face of rising inflation forecasts. In both cases, the ECB did ultimately react to these shocks, but the reaction came too late according to our sufficient macro statistics.

In units of loss function, these "policy misses" represent $\Delta_a = 0.2$ units of foregone welfare. To get a better sense of a 0.2 welfare loss, we can convert Δ_a in in terms of "inflation equivalent variation", similarly to the concept of consumption equivalent (CE) variation in welfare analysis.[23] The idea is to look for a "variation" $\Delta\pi$ such that $\mathbb{E}\mathcal{L}_t(\Delta\pi) = \mathbb{E}\mathcal{L}_t^0 - \Delta_a$ where $\mathcal{L}_t(\Delta\pi) = \sum_{j=1}^{H} \left[(\pi_{t+j} - \pi^*)\,(1 - \Delta\pi]\right)^2 + \lambda(x_{t+j} - x^*)^2$. The inflation equivalent variation $\Delta\pi$ is the percentage reduction in the inflation gap over the next H periods that would generate the same welfare gains as Δ_a. To a first-order in $\Delta\pi$, we get $\mathcal{L}_t(\Delta\pi) =$

[23] CE is the amount of consumption —here the lower inflation gap— that an agent would require to be indifferent between staying in the economy with the baseline policy and the policy under the alternative (here, OPP-improved) policy.

Figure 4: OPP INNOVATIONS FOR EURO AREA MONETARY POLICY



Notes: Top panel: the ECB Deposit Facility Rate. bottom panel: Innovation to the OPP coming from new information (thick line). Shaded areas report the 67 and 95% confidence bands. The green dashed line depicts the average OPP over 1999-2007, and the red dashed line depicts the average OPP over 2007-2021.

$\mathcal{L}_t^0(1-2\Delta\pi)$, such that

$$\Delta\pi \simeq \Delta_a/2.$$

For the ECB over 1999-2023, we found $\Delta_a = 0.2$, such that the welfare gain of a superior ECB policy represents a 10 percent lower (in absolute value) inflation gap for 4 years, or more tellingly a 40 percent lower inflation gap for one year.

8 Conclusion

In this paper we unified the results from a number of recent studies that evaluate macro policy decisions using sufficient macro statistics. First, we disentangled policy evaluation into two separate tasks: time-t policy evaluation and term policy evaluation. The first task is typically performed repeatedly and in real time by policy makers, and the tools that we outlined help the policy maker to correctly calibrate the policy path given the information available at time t. The term policy evaluation results help the policy maker to ex-post evaluate the appropriateness of policy, and allows to highlight inefficiencies in the reaction functions.

These evaluation results are based on a number of representation, or identification, results that allow to represent counterfactual policy decisions in terms of forecasts and impulse responses under some baseline rule. At the moment these results only exists for linear economies where the policy decisions only affect the macro outcomes via the expected policy path. While simple extensions, such as state dependence and time-varying parameters, are easy to accommodate (e.g. Barnichon and Mesters, 2023*b*), handling more complex nonlinearity remains an open topic.

A practical limitation of the sufficient statistics approach is that it requires the identification of all policy shocks at all horizons of the policy path. For most empirical settings this requirement is too strong as only a few shocks can be empirically identified. To improve on this in future work, the identification of policy shocks — most notable at the long end of the policy paths — should take center stage (Caravello, McKay and Wolf, 2024).

A second and less highlighted practical limitation concerns medium- to long-term forecasts, say between 2 to 5 years, which are notoriously difficult (Farmer, Nakamura and Steinsson, 2024). While there exists a large econometric literature that develops macro economic forecasts, most of the work focuses on improving short run (<1 year) forecasts. Unfortunately, the responses of macro variables to policy changes typically take more time to materialize, implying that correctly calibrating policy paths requires accurate medium term forecasts. Improving medium-to long-run forecasting performances is an important task for future research.

Appendix: Sequence space representation and news shocks

In this section, we clarify our use of a sequence space representation and the role played by news shocks. These elements are important for modern policy evaluation methods, yet they are typically not covered in standard macroeconometric coursework. As we will see, the sequence space representation has important benefits in terms of clarity —loosely speaking, turning a dynamic problem into a seemingly static one—.

Sequence space notation

To help understand the sequence space representation and associated notations, we consider a simple example:

$$y_t = \phi y_{t-1} + \upsilon_t \qquad \upsilon_t \overset{iid}{\sim} (0, \sigma^2) \ ,$$

which is the conventional recursive formulation for an AR(1) model with iid errors. The sequence space representation of this model stacks all current and future outcomes in an infinite vector, i.e. $\mathbf{Y}_t = (y_t, y_{t+1}, y_{t+2} \ldots)'$ and represents the model as

$$\mathbf{\Phi Y}_t = \boldsymbol{\upsilon}_t \ ,$$

where

$$\mathbf{\Phi} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ -\phi & 1 & 0 & 0 & \cdots \\ 0 & -\phi & 1 & 0 & \ddots \\ 0 & 0 & -\phi & 1 & \ddots \\ \vdots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{\upsilon}_t = \begin{bmatrix} \upsilon_t \\ \upsilon_{t+1} \\ \upsilon_{t+2} \\ \upsilon_{t+3} \\ \vdots \end{bmatrix} .$$

Note that for simplicity we have set $y_{t-1} = 0$, which is not necessary and will be avoided below by introducing initial conditions. The representation $\mathbf{\Phi Y}_t = \boldsymbol{\upsilon}_t$ has two key benefits: (i) it looks static facilitating easy manipulation[24] and (ii) changes in $\mathbf{\Phi}$ directly document how the entire path of $y_t, y_{t+1}, \ldots$ changes.

In the formulation above the sequence space is written under perfect foresight, i.e. the future shocks are considered observable. While for some exercises this representation is sufficient and convenient, at times we are interested in the model given the information available at time t. Think of a policy maker at time t who is interested in forecasting the path of y. Such policy maker only has information $\mathcal{F}_t = \{\upsilon_t, \upsilon_{t-1}, \ldots\}$.

To define the model given $\mathcal{F}_t$ let $\mathbb{E}_t(\cdot) = \mathbb{E}(\cdot|\mathcal{F}_t)$ be the conditional expectation operator.

[24]Off course we are ignoring many subtle and important aspects of manipulating infinite dimensional maps, but for most of our purposes this will not harm us.

We have

$$\boldsymbol{\Phi}\mathbb{E}_t\mathbf{Y}_t = \mathbb{E}_t\boldsymbol{\upsilon}_t \ ,$$

where $\mathbb{E}_t\boldsymbol{\upsilon}_t = (\mathbb{E}_t\upsilon_t, \mathbb{E}_t\upsilon_{t+1}, \ldots)' = (\upsilon_t, 0, \ldots)'$ as the shocks in this example are iid.

News shocks

In many macro policy settings the exogenous information that is recovered, e.g. using a narrative approach, does not necessarily pertain to the contemporaneous value of the policy instruments. Quite often such exogenous information is also about future values of the policy instruments. Clearly, such information about future policy can affect how agents act today and therefore the release is relevant for policy makers, we refer to these exogenous movements as news shocks.

Concrete examples include a central bank that announces to keep the interest rate low for the coming years, or governments who make plans for spending, taxes and transfers for the coming four years. In each case the exogenous components of such plans can be regarded as news shocks pertaining to the different horizons of the plan. As we will see below such news shocks provide important information that can be used to evaluate policy decisions.

To introduce news shocks in an easy way we build on the previous AR(1) example. Consider

$$y_t = \phi y_{t-1} + \upsilon_t \ , \qquad \upsilon_t = \tilde{\upsilon}_{t-1,t} + \tilde{\upsilon}_{t,t} \ ,$$

where $\tilde{\upsilon}_{t-1,t}$ is the news shock that was released at $t-1$ and contains news about υ_t in time period t, and $\upsilon_{t,t}$ is the contemporaneous shock: released at t about t. We assume that all $\tilde{\upsilon}_{j,t}$ are mutually and serially uncorrelated. The time-t information set $\mathcal{F}_t$ now includes all shocks that are released prior or at time t, i.e. $\mathcal{F}_t = \{\tilde{\upsilon}_{j,k}, j \leq t, k \geq j\}$.

The sequence space representation evaluated given $\mathcal{F}_t$ takes the same general form as above

$$\boldsymbol{\Phi}\mathbb{E}_t\mathbf{Y}_t = \mathbb{E}_t\boldsymbol{\upsilon}_t$$

but now

$$\mathbb{E}_t\boldsymbol{\upsilon}_t = \begin{bmatrix} \mathbb{E}_t\upsilon_t \\ \mathbb{E}_t\upsilon_{t+1} \\ \mathbb{E}_t\upsilon_{t+2} \\ \mathbb{E}_t\upsilon_{t+3} \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \tilde{\upsilon}_{t,t} \\ \tilde{\upsilon}_{t,t+1} \\ 0 \\ 0 \\ \vdots \end{bmatrix}}_{\text{time-}t\text{ news shock}} + \underbrace{\begin{bmatrix} \tilde{\upsilon}_{t-1,t} \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}}_{\text{old news}} .$$

Now there are two shocks that are released at time t: news about time t — $\tilde{\upsilon}_{t,t}$ — and news about $t+1$ — $\tilde{\upsilon}_{t,t+1}$ —. This example, can be generalized by considering

$$y_t = \phi y_{t-1} + \upsilon_t \ , \qquad \upsilon_t = \sum_{j=0}^{\infty} \tilde{\upsilon}_{t-j,t} \ ,$$

where these is now an entire sequence of news shocks. We have

$$\mathbb{E}_t \boldsymbol{\upsilon}_t = \begin{bmatrix} \tilde{\upsilon}_{t,t} \\ \tilde{\upsilon}_{t,t+1} \\ \tilde{\upsilon}_{t,t+2} \\ \tilde{\upsilon}_{t,t+3} \\ \vdots \end{bmatrix} + \begin{bmatrix} \sum_{j=1}^{\infty} \tilde{\upsilon}_{t-j,t} \\ \sum_{j=2}^{\infty} \tilde{\upsilon}_{t+1-j,t} \\ \sum_{j=3}^{\infty} \tilde{\upsilon}_{t+2-j,t} \\ \sum_{j=4}^{\infty} \tilde{\upsilon}_{t+3-j,t} \\ \vdots \end{bmatrix} = \tilde{\boldsymbol{\upsilon}}_t + \mathbf{X}_{t-1} \ ,$$

where $\tilde{\boldsymbol{\upsilon}}_t = (\tilde{\upsilon}_{t,t}, \tilde{\upsilon}_{t,t+1}, \ldots)'$ is the path of time t news shocks and $\mathbf{X}_{t-1}$ captures initial conditions.

What are we identifying?

Having defined the news shocks we clarify how the existing empirically identified macro shocks can be conceptually relate to the news shocks. The short answer is that in most empirical settings we will not know exactly which combination of news shocks is being empirically identified, and the best we can say is that some combination of news shocks is being captured. The good news is that this is fine for most policy evaluation exercises. To make this clear consider the following examples.

Suppose that $\mathbb{E}_t(y_t, y_{t+1}, \ldots)$ is the expected interest rate path and $\tilde{\boldsymbol{\upsilon}}_t = (\tilde{\upsilon}_{t,t}, \tilde{\upsilon}_{t,t+1}, \ldots)'$ is the path of monetary policy news shocks that are announced at time t. In practice, we often use proxies for such shocks that are obtained by measuring changes in asset prices in short windows around press conferences of central banks (e.g. Kuttner, 2001). For the sake of the argument, suppose that these high frequency identified measures are exactly correct, i.e. exogenous and not contaminated by measurement error.

Suppose that the asset used is the short term interest rate, does this make the identified shock the contemporaneous shock, i.e. $\tilde{\upsilon}_{t,t}$? Not necessarily, if the press conference only announces changes in future interest rates then the measured high frequency change in the short term interest rate is driven by some $\tilde{\upsilon}_{t,t+h}$. In fact, in most cases we will not be sure which specific news shocks are responsible for the change as the press conference could be about many horizons of monetary policy.

Similarly, consider the government spending shocks identified by Ramey and Zubairy (2018). The recovered series contains news about military defense spending that is obtained from news paper articles. Similarly as above, the horizon to which the news pertains is often not clear from the articles and therefore we cannot label the shocks as being a specific $\tilde{v}_{t,t+h}$.

In general, close inspection of the main identifying strategies for macro shocks reveals that it is often not possible to determine to which particular horizon the identified shock pertains. We will therefore postulate that the identified shocks are some linear combinations of the theoretically defined news shocks, i.e. we identify some subset

$$\tilde{\boldsymbol{v}}_{a,t} = A\tilde{\boldsymbol{v}}_t$$

where A is a weighting matrix. In the ideal scenario we would like to span the entire path of news shocks and A is some invertible map, yet in practice we often will only have access to a few news shocks.

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